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A MULTI-OBJECTIVE CREDIBILISTIC APPROACH TO PORTFOLIO OPTIMIZATION: BALANCING RETURN, RISK, AND LIQUIDITY IN EMERGING AND DEVELOPED MARKETS

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Abstract

Purpose. The current study will use credibility theory to optimize multi-objective portfolios in the Colombian stock market, taking the US market as a reference.

Methodology. L-R fuzzy numbers are applied to model uncertainty in each asset's future liquidity and return. Cardinality constraints and upper and lower bounds are included for a more realistic model. The analysis is based on a sample of 779 weeks of data between 2009 and 2023 from the Colombian stock exchange, complemented with a comparative evaluation of a portfolio composed of larger-cap NYSE stocks, which shows the model's adaptability in both developed and emerging market contexts.

Findings. The model balances return, risk, and liquidity effectively, hence underlining the main trade-offs that, for example, in the Colombian market, a higher level of liquidity involves higher risk and lower profitability. In contrast, this relation is weaker for the US market, which reflects greater stability. Such analysis of optimal portfolios in both markets outlines structural differences and proves the ability of our model to provide efficient solutions tailored to each market environment.

Originality. This is the first study to apply a multi-objective credibility model in portfolio selection in the Colombian stock exchange. It contributes to the literature on portfolio optimization in important ways: it offers empirical results for an underexplored region, helping academics and practitioners refine investment decision-making in emerging markets.

Keywords: Fuzzy portfolio selection; L-R Fuzzy numbers; Credibility theory; Mean-Semivariance-Liquidity; Colombia stock market.

JEL index: G11

1. Introduction

The Colombian stock market is subject to the great challenges typical of emerging markets, such as volatility and the management of liquidity risks in a high uncertainty environment. Although the Colombian Stock Exchange, BVC, has grown amazingly and plays a substantial role within the national economy, complex economic and political conditions demand more sophisticated investment strategies, coupled with its integration into Latin American markets through the Latin American Integrated Market, MILA (Mellado & Escobari, 2015; Yepes-Henao et al., 2021). In this context, Colombian investors must optimize their portfolios in an environment characterized by high volatility and continuous exposition to external and internal factors affecting both expected returns and assumed risks (Agudelo & Castaño, 2011).

More recently, portfolio selection theory has incorporated approaches that transcend the traditional mean-variance model of Markowitz (1952) in order to respond to the complexities of emerging markets. Among these developments, fuzzy models have highlighted themselves as being particularly relevant in the capturing of intrinsic uncertainty in financial markets with the help of credibility theory and using L-R type fuzzy numbers. Those techniques allow the representation of ambiguity and subjectivity in the estimation of important financial variables, therefore helping make decisions in highly complex and uncertain environments (García et al., 2020; Ruiz et al., 2019). However, a critical review of the literature identified in this study shows that very few applications of such models to emerging markets, and particularly in Colombia, exist, evidencing a research opportunity to explore how these tools may respond to the particular characteristics and challenges these environments present. From this approach, multi-objective portfolio selection models that integrate, along with risk and return, liquidity factors, very important in shallower markets like the Colombian one (Yepes-Rios et al., 2015) must be created. The lack of an approach that considers these elements could notably limit the possibility of investors' optimizing their investment decisions since it affects both stability in markets and growth of capital in the country (Garay & Pulga, 2021).

Based on the above, the question of this research can therefore be stated as: How can a multi-objective credibilistic model, taking return, risk, and liquidity into account, improve portfolio selection in the Colombian stock exchange market, allowing investors to make better decisions under conditions of high uncertainty and volatility? The study then answers this question by introducing a multi-objective credibilistic model, which is among the first applications of this methodology in the South American market. This multi-objective credibilistic model is ideal for emerging markets because it simultaneously handles the three important dimensions, return, risk, and liquidity, through an approach oriented toward maximizing expected return, minimizing semivariance as a measure of risk, and maximizing liquidity. This multidimensional approach responds to the particularities of the Colombian market and provides investors with a tool that allows them to more easily identify portfolios in line with their return, risk management, and liquidity goals in environments with high variability and uncertainty.

The rest of the paper is organized as follows: the literature review underlines some important works on fuzzy portfolio selection; the methodology section details data collection and model formulation; the results and discussion section analyzes the findings of the study; lastly, conclusions summarize the main implications, acknowledge the study's limitations, and provide directions for future research.

2. Literature Review

Among stock market investors, there are numerous means of investing. The popular way is choosing a single firm in which the investor can apply fundamental analysis

(Abuselidze & Slobodianyk, 2021; Laili & Anah, 2023; Wafi et al., 2015) to analyse firms. Other investors apply technical analysis (García et al., 2018). Another strategy in this regard is active management of a diversified portfolio of firms, where the selection is made simultaneously. On the other hand, passive management of portfolios involves strategies that replicate market indices (García et al., 2017; Tokic, 2020; Zaremba, 2015). However, to outcompete the market, some investors actively engage in alternative portfolio selection strategies (García et al., 2022; García, González-Bueno, Oliver, & Tamošiūnienė, 2019; Jalota et al., 2017a; Nurhayati et al., 2021; Oliver, 2021; Orihuel Bañuls, 2021). Integrating fuzzy portfolio selection methodologies offers a compelling and rigorous approach to enhancing the investment decision-making process by effectively addressing uncertainty, imprecision, and ambiguity in financial markets (García et al., 2020; García, González-Bueno, Oliver, & Riley, 2019; Hong et al., 2023; Ruiz et al., 2019). By incorporating fuzzy logic, these methodologies establish a robust framework for decision-making, enabling investors to navigate market complexities and make informed choices.

Portfolio selection involves distributing an investor's wealth among various financial securities to minimize the risk of terminal wealth while achieving a desired expected terminal wealth. Markowitz (1952) pioneered this field with the mean-variance model, using expected portfolio return and variance as return and risk measures, respectively. However, when portfolio returns exhibit asymmetry, variance becomes inadequate as a risk measure, since it penalizes high and low returns equally. This limitation, which arises from the penalization of extreme deviations from expected return, has fostered the development of downside risk measures, including semivariance (Markowitz, 1959), lower partial moment (Bawa, 1975; Fishburn, 1977), semi-absolute deviation (Speranza, 1993), value at risk (VaR) (J.P.Morgan, 1996), and conditional value at risk (CVaR) (Rockafellar & Uryasev, 2000, 2002). Semivariance, in particular, is widely accepted to be a downside risk measure because it ignores returns beyond a threshold, therefore becoming more adequate to represent risk. Markowitz et al. (1993) indicated that semivariance is especially applicable in instances where an investor would be much more concerned about underperformance than with overperformance.

A comprehensive review of the literature on portfolio selection shows that the traditional studies have mostly been carried out with the aim of balancing risk and return, an aspect profoundly developed by García et al. (2015), Gonzalez-Bueno & Chacón-Arias (2014), and Markowitz (1952). However, it is crucial to adopt more dimensions in relation to investor satisfaction. By including the mentioned factors in the process of portfolio selection, portfolios can be identified which, apart from the risk and return objectives, deliver superior performance based on broader criteria. This holistic approach, as espoused by Gupta, Inuiguchi, et al. (2013), makes the scientific and financial bases of portfolio selection stronger and therefore allows investors to come up with optimal portfolios that are aligned with their unique preferences and goals.

Considering the importance of liquidity in investing decisions it is necessary to recognize its place along with return and risk as highlighted in previous studies (Arenas-Parra et al., 2001; García et al., 2020; Jalota et al., 2017a; Kumar et al., 2021; Mehlawat et al., 2020). Investors naturally favor portfolios that have liquid assets that can be easily converted into cash if required. It follows that introducing liquidity as an additional criterion within the mean-semivariance model increases its practical relevance and conformity with investor preferences, thereby yielding a stronger and more rounded portfolio selection. Stock markets are driven by more than just simple randomness.

Traditional studies assume that asset returns are random variables and come up with models which work on the premise that historical performance predicts future performance (Huang, 2009). However, stock markets are related to vagueness and ambiguity, which are usually expressed using linguistic terms such as "high risk," "low profit," and "low liquidity" that prevail in the everyday investment practice by investors (Gupta et al., 2014a; Gupta, Inuiguchi, et al., 2013; Skrodzka, 2021). To handle such vagueness and ambiguity, Zadeh's (1965) fuzzy set theory has been applied for capturing and modelling investor preferences during portfolio investment. Extensive literature discusses the application of possibility measures in portfolio selection (Abdolbaghi Ataabadi et al., 2023; Deng & Chen, 2021; Meyer, 2023; Vercher et al., 2007; Vercher & Bermúdez, 2012), although these measures are not self-dual (Huang, 2008, 2010). In response, B. Liu and Liu (2002) introduced a self-dual credibility measure, an alternative that overcomes the limitations of possibility measures. Subsequently, modelling asset returns has had the credibility measures proposed to reinforce the financial and scientific basis of portfolio selection (Al-Sadat Miraboalhassani et al., 2022; García, González-Bueno, Oliver, & Riley, 2019; Huang, 2006; Jalota et al., 2023; Khan et al., 2024; Kumar & Yadav, 2024; Mandal et al., 2024).

This paper proposes a new method that extends multi objective portfolio selection models to further increase the scientific rigor and practical relevance of fuzzy portfolio optimization. It treats specifically each asset return as an L-R power fuzzy variable, which expands the study in this area. There is, in fact, the portfolio selection model based on L-R fuzzy numbers, so as to directly capture the uncertainty of portfolio returns: as demonstrated by studies such as individual asset return using L-R fuzzy numbers has received very less attention (García et al., 2022; Vercher et al., 2007). An important issue in fuzzy portfolio optimization is finding the optimal shape of the membership functions to better capture the assets' historical performance data.

The Latin American capital markets, like the Colombian stock market, remain largely unexplored empirically despite a huge amount of portfolio optimization model testing across the world's financial markets. Most previous research has focused on markets such as the Tehran Stock Exchange(Ruiz et al., 2019), the National Stock Exchange of Mumbai (India) (Gupta, Mittal, et al., 2013), the Shanghai Stock Exchange (China) (Meng & Zhong, 2021), and the New York Stock Exchange (USA) (García et al., 2022). This paper fills this gap by applying a robust portfolio optimization model tailored to the Colombian stock market, thereby contributing to the literature and facilitating informed investment decisions and potential economic growth in the region.

This paper is an extension of the stochastic mean-semivariance portfolio selection model, where liquidity is a major measure of portfolio performance. In investment portfolio selection, return and liquidity are taken as L-R power fuzzy variables in the decision-making process, which reflects the multidimensional characteristics of portfolio selection. Budget, bound, and cardinality constraints are added into the model to satisfy investor requirements. The optimization aims at maximizing expected return, minimizing semivariance, and maximizing expected liquidity for a given portfolio. This results in an NP-hard constrained multi-objective optimization problem, which cannot be solved using traditional optimization approaches. For this, the Non-dominated Sorting Genetic Algorithm II (NSGA-II) is used as an efficient solution. The proposed method is empirically verified, proving effective and efficient in the decision-making of portfolio management.

3. Methodology

This section develops a detailed methodology to increase the scientific robustness of the research and to firm up its financial and economic contentions. First, the description of data is given, followed by an explanation of the calculation process of L-R fuzzy numbers within the credibilistic framework that quantifies the behavior of return and liquidity. Next, we present the multi-objective portfolio selection model and the application of the NSGA-II algorithm for effective handling of this model. By exploring these methodological aspects in depth, a solid foundation is established, facilitating a robust analysis of the research topic.

3.1. Data description

The Colombian stock market, known as the Bolsa de Valores de Colombia (BVC), holds a central position within the country's financial landscape. As one of Latin America's prominent financial markets, it provides a dynamic platform for trading stocks, bonds, and other securities. With a diverse range of listed companies, from banking to energy and telecommunications, the Colombian stock market offers investors great opportunities to take part in the country's economic growth and development. As a critical site for capital formation and investment, BVC helps in the efficient allocation of resources, enhances liquidity, and contributes to the general stability of the financial system. Through greater integration into global markets and firm commitment to improving market efficiency, the Colombian stock market attracts more and more local and international investors to create wealth and diversify their portfolios.

To really put the MCMSL model in the spotlight as an astoundingly effective model, this study took an empirical approach that will go down in history as a great addition to existing literature. We are going to use a unique dataset from the Colombian stock market in order to explore new avenues in this particular field. The study uses adjusted weekly closing prices and stock liquidity indicators carefully collected for 779 weeks, from January 2, 2009, until December 30, 2023. This deep analysis will surely find insights that could change the view of financial markets forever. According to the World Federation of Exchanges (WFE), by the end of 2023, the Bolsa de Valores de Colombia listed a total of 65 companies. To construct the investment portfolio for this study, two criteria were established for the candidate stocks throughout the study period: (1) consistent weekly trading activity and (2) an average monthly trading volume exceeding the national market average. Applying these criteria, we identified a subset of 19 assets (n=19) that met these conditions and included prominent stocks such as Bogota, Bcolombia, Pfbcolom, Bvc, Celsia, Cemargos, Corficolcf, Pfcorficol, Ecopetrol, Enka, Éxito, Grupoargos, Grupoaval, Gruposura, Isa, Mineros, Nutresa, Pazrio, and Fabricato. Clearly, the selected assets are the benchmark on which to test the MCMSL model and observe its practical applicability and thus contribute to developments in financial research. To further demonstrate the model's generalizability, this study includes a comparison with a portfolio composed of the largest market capitalization stocks listed on the NYSE. This approach not only enriches the analysis but also illustrates the model's adaptability and robustness across different market contexts. This control group of 48 assets (n=48) was chosen based on the same criteria used for the Colombian portfolio for the exact period to ensure the consistency of the evaluation. It comprises major stocks such as Apple Inc., Microsoft Corp., Amgen Inc., Citigroup Inc., ConocoPhillips, Mondelez Intl., Inc., 3M Company, American Intl. Group, Inc., Occidental Petroleum Corp., and Paramount Global. Through the introduction of assets from a mature market, this research will help confirm whether the MCMSL model application is possible for both emerging and developed economies, indicating its potential value as a flexible tool for portfolio optimization.

3.2. Calculation of L-R fuzzy numbers and credibility values

The fundamental concepts of fuzzy numbers, characterized through reference functions L and R, play a central role in this work (L, R:[0,1]→[0,1]). These functions, under certain conditions required by Dubois and Prade (1980), namely L(1) = R(1) = 0, L(0) = R(0) = 1, and the fact that both L(x) and R(x) are strictly decreasing and upper semicontinuous, provide the vehicle for exacting and meticulous analysis in the fuzzy portfolio optimization framework. Following these basic principles, we set a proper base for the discussion of scientific, financial, and economic issues concerning fuzzy numbers within the context of portfolio selection.

The precise structure of the membership function for the L-R fuzzy number $\widetilde{A}=(a, b, c, d)_{L_\pi R_\rho}$ has the following form:

$$\mu_{\tilde{A}}(x)=\begin{cases} L_\pi\left(\frac{b-x}{b-a}\right), & \text{If } a \le x < b \\ 1, & \text{If } b \le x \le c \\ R_\rho\left(\frac{x-c}{d-c}\right), & \text{If } c < x \le d \\ 0, & \text{Otherwise,} \end{cases}$$

In this study, the spreads of the L-R fuzzy number Ã, denoted by (b–a) and (d–c), are thoroughly discussed, while accepting the reference functions of the power family of positive parameters π and ρ, namely $L_\pi(x)=1-x^\pi$ and $R_\rho(x)=1-x^\rho$, introduced by Jalota et al. (2017a). The use of L-R power fuzzy numbers, defined as $\tilde{A}=(a, b, c, d)_{\pi,\rho}$, strengthens the scientific, financial, and economic arguments for the study to focus on fuzzy portfolio optimization.

The credibilistic expected value of the L-R power fuzzy number $\tilde{A}=(a, b, c, d)_{L_\pi R_\rho}$ can be expressed in its crisp equivalent form, obtained by calculating the expected value of a fuzzy variable (Jalota et al., 2017c):

$$E(\xi)=\frac{1}{2}\left[b + c + \frac{\rho(d - c)}{\rho + 1} - \frac{\pi(b - a)}{\pi + 1}\right] \qquad (1)$$

Then, the expression of maximizing the expected return of the portfolio can be represented as follows:

$$\text{Max} = F_1(\omega_i) = \sum_{i=1}^{n}\left[\left[\frac{1}{2}\left[b_{r_i}+c_{r_i}+\frac{(d_{r_i} - c_{r_i})\rho_{r_i}}{\rho_{r_i} + 1} - \frac{(b_{r_i} - a_{r_i})\pi_{r_i}}{\pi_{r_i} + 1}\right]\right]\omega_i\right] \qquad (2)$$

The stock's liquidity indicator serves as the defining measure for the asset's liquidity:

$$\text{Liquidity} = \frac{DOT}{TDP}\left(\sqrt{\left(\frac{NST}{TAS}\right)\left(\frac{TSV}{TAV}\right)}\right) \qquad (3)$$

Where the period for which the observations are made is defined by DOT, number of days for which the stock traded at least once; TDP is the total number of days in the period; NST is the number of stock trades in the period; TAS is the total trades of all stocks in the period; TSV is the trading stock volume in USD in the period, and TAV is the trading volume of all stocks in USD in the period. Hence, a specific objective for the portfolio's expected liquidity performance maximization can be defined as follows:

$$\text{Max} = F_2(\omega_i) = \sum_{i=1}^{n}\left[\left[\frac{1}{2}\left[b_{l_i} + c_{l_i} + \frac{(d_{l_i} - c_{l_i})\rho_{l_i}}{\rho_{l_i} + 1} - \frac{(b_{l_i} - a_{l_i})\pi_{l_i}}{\pi_{l_i} + 1}\right]\right]\omega_i\right] \quad (4)$$

The estimation of portfolio risk is done using the semivariance measure, hence the objective of minimizing the portfolio's semivariance can be represented as(Jalota et al., 2017c):

$$\text{Min} = F_2(\omega_i) = \begin{cases} \frac{(e-a_{r_p})^2}{2} - \frac{(e-a_{r_p})(b_{r_p}-a_{r_p})}{\pi_{r_p}+1} + \frac{(b_{r_p}-a_{r_p})^2}{(\pi_{r_p}+1)(\pi_{r_p}+2)} + \frac{(e-c_{r_p})^{\rho_{r_p}+2}}{(d_{r_p}-c_{r_p})^{\rho_{r_p}}(\rho_{r_p}+1)(\rho_{r_p}+2)}, & \text{If } c_{r_p}<e\leq d_{r_p} \\ \frac{(e-a_{r_p})^2}{2} - \frac{(e-a_{r_p})(b_{r_p}-a_{r_p})}{\pi_{r_p}+1} + \frac{(b_{r_p}-a_{r_p})^2}{(\pi_{r_p}+1)(\pi_{r_p}+2)}, & \text{If } b_{r_p}<e\leq c_{r_p} \\ \frac{(e-a_{r_p})^2}{2} - \frac{(e-a_{r_p})(b_{r_p}-a_{r_p})}{\pi_{r_p}+1} - \frac{(b_{r_p}-e)^{\pi_{r_p}+2}}{(b_{r_p}-a_{r_p})^{\pi_{r_p}}(\pi_{r_p}+1)(\pi_{r_p}+2)} + \frac{(b_{r_p}-a_{r_p})^2}{(\pi_{r_p}+1)(\pi_{r_p}+2)}, & \text{If } a_{r_p}<e\leq b_{r_p} \\ 0 & \text{Otherwise} \end{cases} \quad (5)$$

The research focus lies in the application of the multiobjective credibilistic mean-semivariance-liquidity portfolio selection model, which focuses on maximizing the portfolio's return (Max F1 (ωi)), maximizing the portfolio's liquidity (Max F2 (ωi)), and minimizing the portfolio's risk (Max F3 (ωi)). Besides these objectives, this model considers some real-world constraints, such as budget allocation over the whole budget, an upper bound on investment, limitations of the number of assets in the portfolio, and no short selling.

The capital budget on the assets can be formulated as

$$\sum_{i=1}^{n} \omega_i = 1 \quad (6)$$

The constraint against short selling assets is written as

$$\omega_i \geq 0, \quad i=1, 2, \ldots, n \quad (7)$$

The maximum fraction of capital that can be invested in any single asset is called

$$\omega_i \leq u_i y_i, \quad i=1, 2, \ldots, n \quad (8)$$

The smallest amount of capital that can be invested in a single asset is called

$$\omega_i \geq l_i y_i, \quad i=1, 2, \ldots, n \qquad (9)$$

The quantity of assets held in the portfolio is stated as:

$$\sum_{i=1}^{n} y_i = k, \qquad (10)$$
$$y_i \in \{0,1\}, \quad i=1, 2, \ldots, n$$

Within this research, an efficient method of portfolio evaluation is to find an optimum portfolio P such that there does not exist another portfolio P' for which the conditions $P_{F1(\omega i)} \geq P'_{F1(\omega i)}$ and $P_{F2(\omega i)} \geq P'_{F2(\omega i)}$ and $P_{F3(\omega i)} \leq P'_{F3(\omega i)}$ are fulfilled with strict inequality holding for at least one criterion. The resulting set of Pareto-efficient solutions, in the given decision space, defines the Pareto optimal set, which generally forms a frontier known as the Pareto optimal frontier. Portfolios lying on this frontier are considered non-dominated, in the sense that no portfolio on the frontier can outperform all others simultaneously with respect to the chosen criteria of return, risk, and liquidity.The optimization process in multi-objective portfolio selection model deviates from the traditional quadratic optimization approach, since it is a complex quadratic mixed-integer problem classified as NP-hard. In order to deal with this complexity, the use of MOEAs is particularly effective. Here, we use the Non-dominated Sorting Genetic Algorithm II NSGA-II (Liagkouras & Metaxiotis, 2015) originally proposed by Deb et al. (2002). Further explanation of the structure of the algorithm is given by the studies of Palanikumar et al. (2009) and Deb et al. (2002). The parameters of experimental configuration used in this study are population size 400, distribution index 10 for crossover, probability of crossover 0.9, distribution index 50 for mutation, probability of mutation 0.01, and the maximum number of generations 2000.

4 Results and discussion

We compute the asset returns, rit, by the formula $r_{it}=(p_{it}-p_{it-1})/(p_{it-1})$, i = 1, 2, …, 19; t = 1, 2, …, 779 where pit denotes the closing price of i-th asset on Friday of week t. To represent the uncertainty of returns, the membership function of the L-R fuzzy return (ξ_{l_i}) is deduced from returns data on an empirical basis, and particularly through the use of percentiles. In the same way, the membership function of liquidity (ξ_{l_i}) is obtained from stock liquidity indicators, for which methodology presented by (Vercher, 2008; Vercher & Bermúdez, 2012, 2013, 2015) can be followed. By using these fuzzy measures, extensive analysis of the performance and liquidity of assets becomes possible, improving decision-making in portfolio optimization and selection. The diversification parameters suggested in Gupta et al. (2014b), with li= 0 and ui= 0.3 for all i = 1,2, …, 19, are our benchmark for taking the diversification concept into consideration while constructing a portfolio. As confirmed in the literature, optimal diversification is typically achieved within a given range of assets; Gupta et al. recommend portfolios composed of between 3 and 10 assets. Similar to this principle, our study focuses on constructing an admissible portfolio composed of k = 10 judiciously chosen assets to harness the full benefits of diversification effectively.

Figures 1 and 2 present the three-dimensional visualizations of the final portfolios obtained by NSGA-II for the MCMSL model applied to the Colombian and U.S. stock markets, respectively. Each cluster of data points represents a set of non-dominated solutions or efficient portfolios with optimal trade-offs between return, risk, and liquidity. The visualization graphically illustrates the inherent trade-offs between these objectives, in that improving one often requires concessions in another.

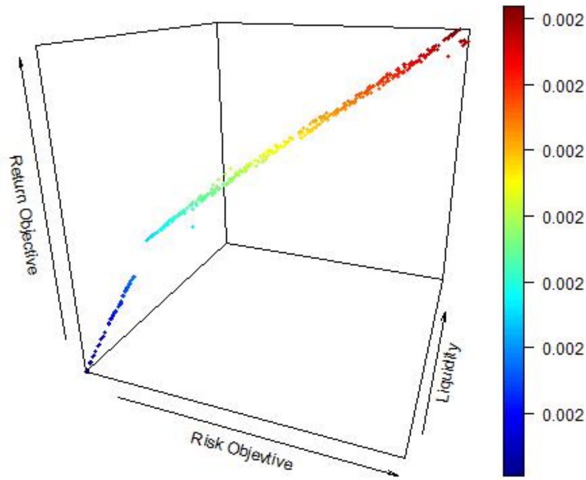


Figure 1. Pareto Optimal Frontier of the Colombian MCMSL Model
Source: *The authors*

Figure 1 depicts the Pareto optimal frontier for the Colombian market; the distribution of solutions is relatively linear along the dimensions of return, risk, and liquidity. This would suggest a balanced but constrained trade-off where the rise in return is associated with an adjustment in risk and liquidity. This is consistent with previous studies on emerging markets, indicating that market depth is limited and volatility higher, factors that require careful balancing between risk and return (Agudelo & Castaño, 2011; Yepes-Henao et al., 2021). The nearly connected Pareto front of the Colombian model indicates that the NSGA-II algorithm efficiently identifies portfolio options that meet diverse investor preferences under the structural constraints of an emerging market, in which interdependencies between risk and return, and liquidity, are more pronounced (Ruiz et al., 2019).

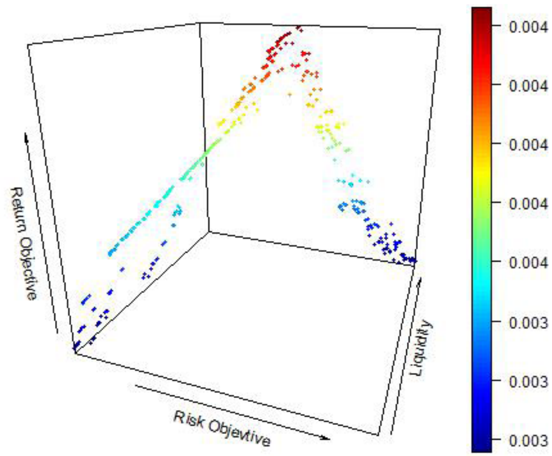
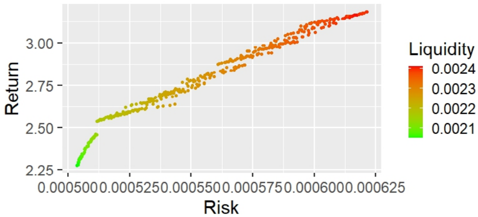
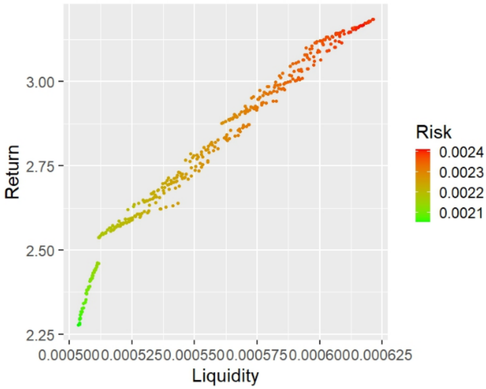


Figure 2. Pareto Optimal Frontier of the U.S. MCMSL Model
Source: *The authors*

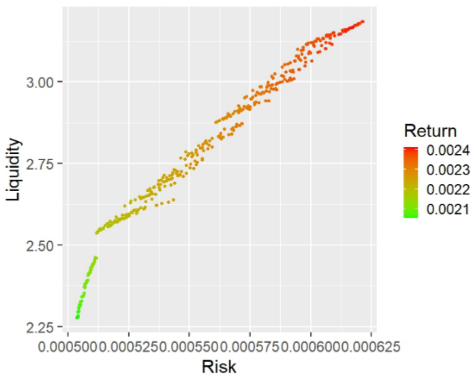
By contrast, Figure 2 shows the Pareto optimal frontier for the U.S. model, which has a more spread out efficient portfolio distribution, especially on the extremes of the trade dimensions. This points to a wider range of potential solutions in the U.S. model, indicating greater flexibility in reaching varying levels of return, risk, and liquidity, which accords with findings in developed markets where greater market depth and liquidity provide more options for investments. Developed markets thus enable optimal returns with less sharp trade-offs in risk and liquidity (Gupta et al., 2020; Hong et al., 2023). Most of the literature supporting this view shows that stability and efficiency in developed markets generally allow for more flexibility and detail in the risk-return relationship (Abuselidze & Slobodianyk, 2021; García et al., 2022). Our results build on existing research in portfolio optimization by underlining liquidity as an essential dimension to consider alongside return and risk in portfolio optimization, more so within emerging markets (Arenas-Parra et al., 2001; Jalota et al., 2017b). By including liquidity as a key selection criterion, the MCMSL model becomes very useful for capturing the trade-offs inherent in each market environment. In Colombia, where market depth is lower, liquidity constraints play a more important role, since less liquid assets are more subject to increasing transaction costs and volatility. In contrast, the U.S. market benefits from higher liquidity, allowing investors to focus more on optimizing returns and managing risk.



(a) Expected Return vs. Downside Risk



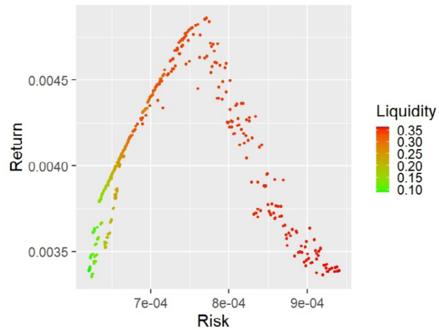
(b) Expected Return vs. Expected Liquidity



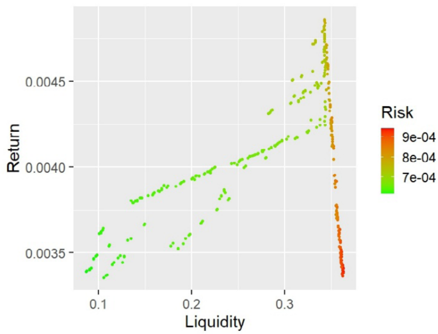
(c) Expected Liquidity vs. Downside Risk

Figure 3. Objective Function Values for All Solutions Generated by NSGA-II for the Colombian Market

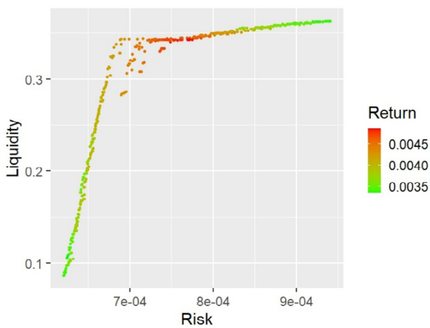
Source: *The authors*



(a) Expected Return vs. Downside Risk



(b) Expected Return vs. Expected Liquidity



(c) Expected Liquidity vs. Downside Risk

Figure 4. Objective Function Values for All Solutions Generated by NSGA-II for the U.S. Market

Source: *The authors*

From a practical point of view, it would be important to give more insights into the behavior of each pair of objectives in the MCMSL model. Therefore, as can be seen in Figures 3 and 4, for the Colombian and USA markets respectively, bi-dimensional representations are shown for each pair of objectives. The visual analysis of these results demonstrates the effectiveness of the proposed approach in generating adequate Pareto optimal fronts for both markets. It can be noticed from Figure 3, which refers to the Colombian market, that there is a clear trade-off between liquidity and risk: higher liquidity implies higher risk with lower returns. This agrees with the peculiarities of emerging markets, like Colombia, whose structural limitations of market depth, restrictions in the trading volumes, and reduced numbers of listed companies imply increasing costs of liquidity. Previous literature in emerging markets has documented this phenomenon, where investors usually demand higher returns to compensate for the extra risks arising from the limited liquidity (García et al., 2015, 2020). A thin market, as in the case of Colombia, means that price changes brought on by large trades would then make portfolios tilted towards liquidity inherently riskier. Figures 3b and 3c demonstrate this in the following, representing a fundamental balance in return optimization with minimum risk and maximum liquidity. For instance, highly liquid portfolios represent high return but also significant risk; portfolios where liquidity is restricted also represent reduced risk but sometimes high returns.

By contrast, Figure 4, representing the U.S. market, reveals clear separation by goals. Figure 4a, the relationship between liquidity and risk is less pronounced than in the case of the Colombian model, with the liquidity levels spread over a broader range of risk-return profiles. This dispersion most likely reflects the greater depth and efficiency of developed markets such as the U.S., where large trading volumes coupled with a greater investor base contribute to greater market stability. Hence, liquidity in such markets may walk hand in hand with profitability without excessive risk, complementing findings in developed market studies that underline nuanced risk-return optimization (Jalota et al., 2017a; Kumar et al., 2021; Mehlawat et al., 2020). Figure 4c provides even more evidence to back this up, showing a wider variety of trade-offs between return, risk, and liquidity in the U.S. market, where market stability and investor confidence allow for less rigid connections between risk and liquidity than in emerging markets.

A comparison between Figures 3 and 4 shows that there are critical differences in the structure of the Pareto optimal fronts for the Colombian and USA markets. The Colombian market's more constrained trade-off structure reflects the challenges inherent in emerging markets, where liquidity and risk are closely linked due to structural limitations. These findings underscore the importance of developing portfolio optimization models tailored to the specific characteristics of emerging markets, recognizing the heightened sensitivity of Colombian stocks to liquidity constraints and risk factors.

Table 1 reports the Spearman partial correlations between return, risk, and liquidity in the Colombian and U.S. markets, where the main differences lie in how these variables interact with each other within each environment. In the Colombian market, there is a strong positive correlation between the return and risk variables, compatible with the characteristics of an emerging market, where investors expect higher returns to compensate for bearing additional risk. This pattern shows that the relationship between risk and return is much stronger due to the higher volatility and shallower market depth. In contrast, the U.S. market reveals a far lower correlation degree for these variables, implying that in a developed market, the concepts of risk and return would not be so directly linked. Higher stability and efficiency of the U.S. market allow investors to manage risk independently of return to a greater extent. The relationship between liquidity and risk is also stronger for the U.S. market compared to Colombia. This difference may thus be an indication that US liquid assets are more sensitive to market movements, impelled by high institutional participation and very fast price adjustments. This relation is also relevant in Colombia, though the impact of liquidity on risk is weaker, possibly due to the fact that in this market, there are lower market depths and less trading activity. Moreover, the negative relationship between return and liquidity is more pronounced for the Colombian than for the U.S. market. This pattern shows that, in Colombia, more liquid assets bear lower returns, possibly because in times of greater uncertainty, investors prefer liquid assets and thus the expected return is depressed. In the U.S., this relationship is much weaker, as high liquidity is a structural characteristic, not as big a factor in differentiating returns. These differences underline the sway of market context, whether emerging or developed, on the relations between return, risk, and liquidity.

Table 1. Spearman's Rank Partial Correlation Matrix for the Colombian and U.S. Markets

	Return $_{(COL)}$	Risk $_{(COL)}$	Liquidity $_{(COL)}$
Return $_{(COL)}$	1.000		
Risk $_{(COP)}$	0.948**	1.000	
Liquidity $_{(COP)}$	-0.675**	0.874**	1.000

	Return $_{(USA)}$	Risk $_{(USA)}$	Liquidity $_{(USA)}$
Return $_{(USA)}$	1.000		
Risk $_{(USA)}$	0.152**	1.000	
Liquidity $_{(USA)}$	-0.137**	0.992**	1.000

Note: *** Correlation is significant at the 0.01 level*

Source: *The authors*

This study proposes a novel multiobjective framework for portfolio optimization that captures considerations related to return, risk, and liquidity. Most practical investment scenarios involve investors in the challenge of choosing a portfolio on the Pareto optimal front, as depicted in Figures 1 and 2, which can best satisfy their personal preferences and investment goals. To make this selection, the study uses the Sortino ratio, defined as one of the most accepted measures when it comes to evaluating the risk-adjusted return of investment assets or portfolios. Extending the previous application of the Sortino ratio to a credibilistic environment enhances the abilities of the measure in the complexities and uncertainties of the portfolio decision-making process (García et al., 2020; García, González-Bueno, Oliver, & Tamošiūnienė, 2019). In this credibilistic framework, the Sortino ratio is defined as follows:

$$\text{Sortino Ratio} = \frac{E\left(\xi_p\right) - E\left(\xi_{Rf}\right)}{SV(\xi_p)} \quad (11)$$

The expected fuzzy portfolio return, the fuzzy semivariance, and the target rate of return or the desired rate of return will be denoted by , and respectively. In this work, will be taken as the US 1-Year Treasury Constant Maturity Rate.

Table 2. Comparison of Colombian Mutual Funds and Optimally Selected Portfolios

	Return	**Liquidity**	**Risk**	**Beta**
Optimal Portfolio $_{(COL)}$	0.034	7.456	0.202	0.752
Optimal Portfolio $_{(USA)}$	0.012	7.160	0.023	1.013
Colombia A Shares Mutual Fund	- 0.053	5.206	0.184	0.956
Colombia C Equity Mutual Fund	- 0.063	3.966	0.176	0.906
Colombia Class A Shares Mutual Fund	- 0.070	7.686	0.186	0.994
Colombia Class A Equity Mutual Fund	- 0.071	7.126	0.177	0.899

Source: *Economatica*

It is wise to compare the performance of an Optimal Portfolio constructed based on our research model with other investment alternatives available in the market. Table 2 compares the Colombian Optimal Portfolio with four representative mutual funds from the local market and the Optimal Portfolio for the U.S. market. This exercise serves to illustrate in practice the model and highlights the efficiency of a methodology that combines multiple objectives in very different market environments. The Colombian optimal portfolio, for the first half of 2024, shows a return of 3.4%, competitive performance, albeit with a slight difference below some mutual funds. However, the approach is balanced, considering risk and liquidity management. Its liquidity measure, at 7.456, allows trading with an insignificant price impact, a very vital property in market trading that's relatively thin, like Colombia's. Looking at the risk, the Colombian Optimal Portfolio has a value of 0.202, similar to some local mutual funds like the Colombia A Shares Mutual Fund (0.184) and the Colombia Class A Shares Mutual Fund (0.186).

This moderate risk profile is further supported by the Beta coefficient of 0.752, indicating controlled sensitivity to market fluctuations and achieving a balance between stability and adaptability. The use of semi-variance as a downside risk measure now allows a more precise look into potential losses, which appeals to investors with a lower tolerance for negative volatility. Conversely, the U.S. Optimal Portfolio has a lower return of 1.2% but with substantially lower risk (0.023) and high liquidity (7.160). Its Beta coefficient of 1.013 reflects higher sensitivity to movements in the U.S. market, which more than confirms the conservative profile typical for developed markets where stability and reduced volatility are overemphasized. This comparison only underlines the necessity of working out optimization strategies with respect to specific market contexts: for emerging markets, investors may benefit from a risk-return profile inclusive of liquidity considerations; for developed markets, stability often becomes the prime focus. As evidenced, the Colombian Optimal Portfolio is an attractive option for investors looking to balance return, risk, and liquidity in a high-uncertainty environment. With this multi-objective model, one is able not only to deal with portfolio optimization on a global scale but also to come up with practical applications by offering tools adapted to the particular characteristics that each market has. This flags the importance of a flexible strategy able to represent characteristic trade-offs between return, risk, and liquidity.

5. Conclusions

This study presents a multi-objective credibilistic model of portfolio selection, applied innovatively to the Colombian stock market in order to address return, risk, and liquidity within the framework of an emerging market. Using L-R fuzzy numbers and credibility theory, this model is able to capture both types of inherent uncertainty and a characteristic liquidity constraint addressed in the Colombian market.

To further demonstrate the model's versatility and robustness, a comparative analysis was performed with an optimal portfolio composed of the largest market capitalization stocks listed on the NYSE. This not only enriched the analysis but also underlined the model's adaptability in different market contexts. In the Colombian market, the model followed an intuitive trade-off between liquidity and risk: higher liquidity means increased risk and lower returns. This relationship is less pronounced in the U.S. market, where greater market depth and stability reduce these trade-offs. Application of the NSGA-II algorithm made it possible to generate Pareto fronts reflecting these structural differences, providing efficient solutions tailored to each market environment.

The advantages of the proposed optimal portfolio are compared to traditional investment alternatives available in both the Colombian and U.S. markets. It presents relatively competitive returns, a moderate risk profile, and high liquidity, so this portfolio could be very attractive for investors looking for balance and stability in investment options in uncertain environments.

Several limitations of the present study should be noted. The focus on the Colombian market may reduce the potential generalizability of the findings to other emerging and developed markets with different structural characteristics. Future research is therefore encouraged to replicate the model in other emerging markets and test its performance under conditions of heightened instability. Further research might also consider alternative liquidity and risk measures, such as CVaR, and incorporate factors reflecting real investment conditions, like transaction costs, to enhance the model's relevance and accuracy for practice.

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