ISSN 1822-8038 (online)
INTELEKTINĖ EKONOMIKA
INTELLECTUAL ECONOMICS
2024, No. 18(1), p. 80–105

INVESTIGATION OF MODERN INVESTMENT OPPORTUNITIES WITH CRYPTOCURRENCY MARKET: OPTIMIZATION APROACH

Natalia Chernova
National Technical University, Kharkiv Polytechnic Institute, Ukraine
Natalia.Chernova@khpi.edu.ua
ORCID: 0000-0002-0073-8457

Olena Serhiienko
National Technical University, Kharkiv Polytechnic Institute, Ukraine
Elena.Sergienko@khpi.edu.ua
ORCID: 0000-0002-9796-9218

Mykhailo Bril
Simon Kuznets Kharkiv National University of Economics, Ukraine
mihail.bril@hneu.net
ORCID: 0000-0001-6529-7747

Oleksandr Bilotserkivskyi
National Technical University, Kharkiv Polytechnic Institute, Ukraine
Alexander.Belotserkovsky@khpi.edu.ua
ORCID: 0000-0003-4707-7964

Valeriia Kochorba
V. N. Karazin Kharkiv National University, Ukraine
v.y.kochorba@karazin.ua
ORCID: 0000-0002-5509-680X

DOI: 10.13165/IE-24-18-1-04

Abstract

Purpose. The research aim is to study effects of cryptocurrencies inclusion into an investment portfolio. To achieve the aim, portfolios with different composition of traditional and crypto assets are to be formed and compared.

Methodology. It is proposed to conduct the following steps of the appropriate algorithm of investigating modern opportunities with cryptocurrency market: 1) preliminary analysis of the relationships and interdependencies between traditional assets and crypto assets; 2) formation of the initial set of crypto assets that can be potentially included into portfolio; 3) efficient frontier assessment for portfolios with different initial composition of assets; 4) comparative assessment and result analysis.

Findings. The algorithm was implemented for the initial set of five most common traditional assets and ten cryptocurrencies. The latter constituents list was formed according to the value of market capitalization. Than the initial set of crypto assets was reduced according to their multidimensional distances from traditional assets. The results obtained allow to conclude that there are opportunities of portfolio efficiency increase via crypto assets inclusion in its structure. The increase value varies noticeably and depends on the particular kind of crypto assets, their total share and number.

Originality. This research suggests to conduct additional preliminary procedures to choose potential candidates to be included into the traditional portfolio among initial set of crypto assets. Firstly, market capitalization value and low correlations with traditional portfolio constituents are taken into account. Secondly, all assets are presented as points in two-dimensional risk-return space and crypto assets are chosen according to the multidimensional distance measure value.

Keywords: cryptocurrency, investment, portfolio, risk, return, optimization.

JEL Index: G11, C58, C61, O16, G15

Introduction

In recent time investors more often tend to include cryptocurrency assets as additional constituents of their portfolios, arguing their choice by significant changes in cryptocurrency market liquidity, and also large opportunities in gaining yield. Such opportunities list include but not limited by the following items: the speed of making payments, which is ensured by the absence of intermediaries; low transaction fees; the possibility of making transactions at any time, regardless of working hours and days of the week; the possibility of making direct settlements with foreign partners, reducing the costs of converting foreign currency into national currency and vice versa; the possibility of taking into account the inflation risk inherent in fiat currencies; ensuring the confidentiality of agreement participants personal data.

The structure of cryptocurrency market becomes more and more complicated. From it start with Bitcoin as the only element now we can observe nearly 10.87 thousand of crypto assets (Coinbase). Cryptocurrencies can be classified according to many different characteristics. One of the most often applied classification divides cryptocurrencies universe on coins and tokens. Coins are digital assets that are native to their own blockchain (Bitcoin's blockchain coin is BTC, Ethereum's blockchain coin is ETH, Litecoin's blockchain coin is LTC). They are independent of other chains and cannot be used on other chains in their native form. Their primary functions are: to be a store of value and to be a medium of exchange. Tokens are created on blockchains that already exist and offer a broader range of functionalities. They can represent an asset or provide the holder a specific service or access to an application. Tokens can be additionally classified on utility tokens, security tokens, non-fungible tokens. Tether (USDT) and USD Coin (USDC) are examples of tokens on Ethereum.

As for market capitalization distribution there are undoubted leaders such as Bitcoin (51%) and Ethereum (18%), followed by Tether (6%), BNB (2.7%) and XRP (2.5%). All other assets market shares are less than 2%(Coinbase).

A completely natural development of the cryptocurrency market was the appearance of derivatives tied to cryptocurrency, such as futures, options and perpetuals. Crypto derivatives work like those ones at traditional financial markets and provide greater flexibility for market participants. The first Bitcoin futures contracts were listed on CBOE in 2017 but soon were discontinued. The Chicago Mercantile Exchange (CME) also introduced Bitcoin futures contracts in 2017. The contracts trade on the Globex electronic trading platform and are settled in cash. Bitcoin and Ether futures are based on the CME CF Bitcoin Reference Rate and the CME CF Ether Reference Rate.

Nowadays the share of derivatives is much greater than the spot one. As of March 2023 according to Shaun Paul Lee (2023), crypto derivatives part accounted for 74.8% of crypto's total trading volume.

The largest crypto exchange Binance had daily derivative trading volume near $ 679,375,985.79, top 5 assets that formed this market volume were BTC (27.95%), USDT (17.91%), BNB (14.85%), ETH (8.28%) and WBETH (1.87%) as of early April, 2024(CoinMarketCap).

Nowadays investors' possibilities to get direct exposure to the cryptocurrency market are still restricted in many countries and one way to cope with this problem is to invest in crypto ETFs. As of February, 2024 according to Barchart there were 26 crypto ETFs with total assets under management value of nearly $ 52,732,731,600.00. The largest share of nearly 43.28% is accounted for Grayscale Bitcoin Trust (GBTC). Other members from the top of rating with valuable shares are: Ishares Bitcoin Trust (27.13%), Fidelity Wise Origin Bitcoin Fund (17.93%), Proshares Bitcoin Strategy ETF (5.37%) and Bitwise Bitcoin ETF (4.49%).

At the same time, the share of cryptocurrency among other asset classes (equities, bonds, commodities) is still the smallest. According to the Table 1, we see that in terms of capitalization, cryptocurrency occupies a very small part of the global market. However, if comparing growth rates, one can see a significant advance of cryptocurrencies. The cryptocurrency market has skyrocketed over the past 10 years. According to CoinMarketCap as for the core crypto asset Bitcoin its market capitalization has demonstrated total growth of approximately 448% (from $1.59B on 4/09/2013 to $712.60 B on 10/11/2023).

Table 1. Market Capitalization

Asset class	Market Capitalization, trillions USD
Global Fixed Income Market	129.8 (Sifma)
Global Equity Market	101.2 (Sifma)
Precious metals market(gold)	12.2 (IGWT Report)
Bitcoin	0.323(IGWT Report)

According to Fig. 1, Bitcoin performed enormously in 2016, 2018, 2019, 2021 in comparison to other financial asset classes such as stocks, bonds, currencies and commodities.

The yearly growth rates for all mentioned assets except Bitcoin have never exceeded 150% level. In contrast, one can see periods when Bitcoin price demonstrated huge changes. The significant volatility of Bitcoin makes it necessary to conduct comparative analysis of cryptocurrencies and all other assets in the risk-return space. Here we estimate risk and return measures as the standard deviation and the mean of initial time series growth rates.

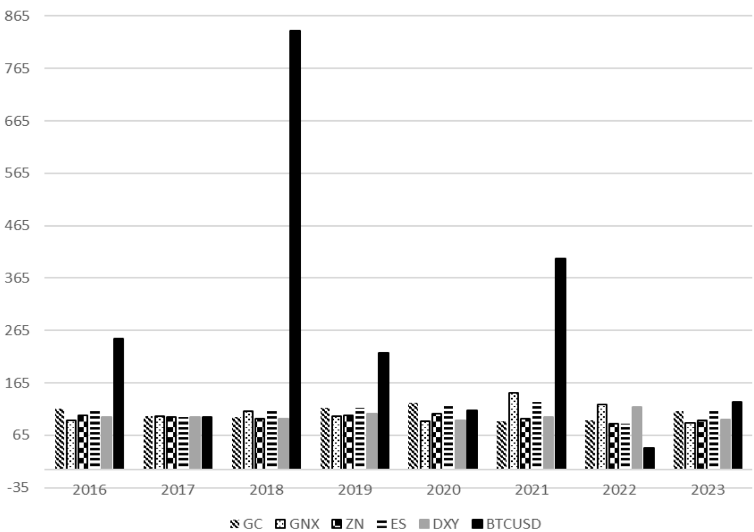


Figure 1. Yearly growth rates, %, GC – gold, GNX – commodities, ZN – bonds, ES – stocks, DXY – dollar index, BTCUSDT – Bitcoin (Calculated by the authors)

Fig. 2 presents so cold traditional assets that were analyzed previously on Fig. 1 and also top ten crypto assets chosen by the value of market capitalization as for the end of October, 2023 (they are: Bitcoin(BTCUSD), Ethereum(ETHUSD), Tether(USDTUSD), Binance Coin(BNBUSD), XRP(XRPUSD), Cardano(ADAUSD), Dogecoin(DOGEUSD), Solana(SOLUSD), Tronix(TRXUSD), Litecoin(LTCUSD)).

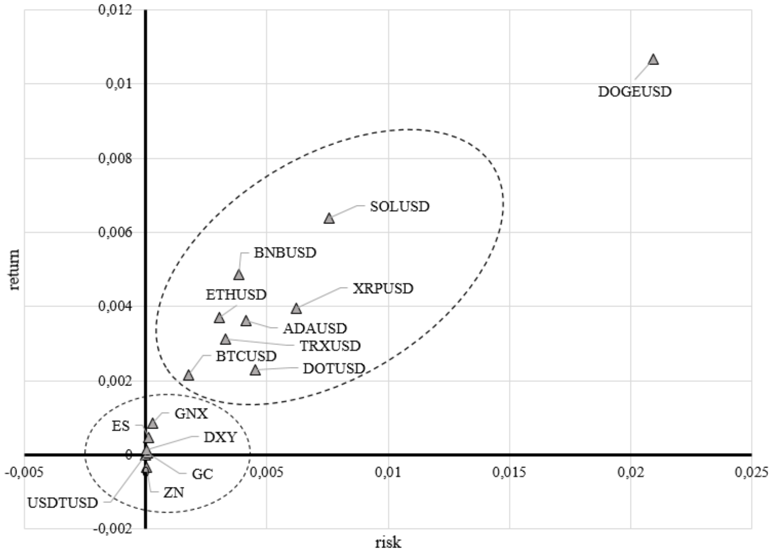


Figure 2. Different assets in risk-return space (Calculated by the authors)

Here we can see two rather homogenous subsets: traditional assets and crypto assets. All traditional assets are situated rather compact near the center of the coordinate system. Depending on the particular cryptocurrency, there are such ones that are relatively further away or relatively closer to the traditional assets universe. The crypto subset has two outliers. The first one is USDTUSD. Because of the fact that it is a stablecoine pegged to the U.S. dollar, it is situated very close to its underlying DXY(dollar index) and in fact is a member of traditional subset. The second outlier is DOGEUSD. It has enormously high levels of risk and return in comparison with all assets.

Thus, cryptocurrency market and its instruments are of high interest as for portfolio investment, and there are prerequisites that including cryptocurrencies in an investment portfolio can be effective.

It is known that investment portfolios can be classified by the total accepted risk value and the level of total return, so the problem to solve can be formulated as follows: is it possible to significantly improve portfolio core characteristics via inclusion of crypto assets in it?

Literature review

General features of cryptocurrency as an investment object have been hardly studied in the variety of scientific researches. Holovatiuk, O., (2020), Ankenbrand, T., & Bieri, D. (2018), Corbet, S., Lucey, B., Urquhart, A., Yarovaya, L., (2019) have investigated the problem of considering cryptocurrencies as a new class of assets. Most of them made the same positive conclusion based on arguments such as correlation among different crypto assets, absence of correlation with external groups of assets, increasing liquidity, growing interest of public authorities, implementation into multiple industries, crypto market total dependence on global shocks and speculations.

In general, researches of cryptocurrencies as participants of an investment portfolio can be divided into two groups. The first group of works study opportunities of constructing mixed portfolios, which consists of cryptocurrencies and some other kinds of assets. The second group of researches dedicated to questions of investing purely in cryptocurrency and comparisons of so cold traditional portfolios and crypto portfolios by their performance.

As for the first group, here crypto assets are often researched as means of diversification. Many authors choose Bitcoin as the only representative of the whole cryptocurrency universe and investigate its diversification abilities. Bitcoin is historically the first cryptocurrency, it has the highest market capitalization value and occupies nearly half of the total market. As a result, Bitcoin and its derivatives are traded on the most famous exchanges, it is characterized by the suitable liquidity values, its historical prices series are often available for free and have the largest length. All those facts make Bitcoin rather ideal object of economic and statistical researches.

Some authors researched the effect of adding Bitcoin into already well diversified portfolios. Usually they were such ones which had stocks, bonds, commodities and currencies in their structure. Also geographical aspect was taken into account by some researchers when such so cold traditional portfolios differed from each other by country specific or geographically specific assets. For instance, Kajtazi, An. & Moro, A. (2019) have distinguished Bitcoin's role in portfolios which had USA, European and Chinese assets as their constituents. Colombo, J., Cruz, F., Paese, L. and Cortes, R. (2021) have researched twenty-one developing and developed country specific portfolios formed of stocks, bonds, real estate, and commodities. Jiaqi Qin, Shansong Huang, Boying Yang, Yilin Ma, Zheng Tao, Shuqi Chen (2022) have investigated only stocks portfolios which consist of stocks of leading companies in different industries.

Damianov, D. S., Elsayed, A. H. (2019) have studied impact of Bitcoin on portfolio which consist of ten global industry sectors stocks. They showed rather significant effect in case of constructing portfolios that maximize Sharpe ratio.

Ma, Yechi & Ahmad, Ferhana & Liu, Miao & Wang, Zilong, (2020) have conducted comparative research of diversification effect of Bitcoin and Ethereum, and also provided results of portfolio diversification with multiple crypto assets. They have presented result for four different initial portfolios: stocks of technological companies, stocks of top-performing companies, currency exchange rates in dollars against five currencies, and commodities.

Such authors as Brauneis, A. & Mestel, R. (2019), Elendner, H., Trimborn, S., Ong, B. & Teik Ming Lee (2016) have investigated the effects of adding not one but several crypto assets into investment portfolio.

Some authors used aggregated cryptocurrency indices in their studies. For instance, Asanga Jayawardhana & Sisira R N Colombage (2023) have applied Bloomberg Galaxy Cryptocurrency Index (BGCI) as a proxy for the crypto market and investigated relationships between BGCI and some indices that represent debt and equity markets.

Such authors as Lorenzo, L., Arroyo, J. (2023), Maghsoodi, Abtin Ijadi, (2023); Elendner, H., Trimborn, S., Ong, B. & Teik Ming Lee (2016) researched opportunities of investing purely in cryptocurrency. Chen, Tian (2021) has examined performance of ten largest cryptocurrencies portfolio and compare portfolio performance with those ones of individual cryptocurrencies. The final conclusion states that individual assets outperform complex portfolio and it is suggested to add the better performing cryptocurrency portfolio to the traditional assets portfolio and examine the performance. Lorenzo, L., Arroyo, J., (2023) have applied clustering method to partition the cryptocurrency space and the automatic selection of the partition that best suits the risk-aversion preference of the investor. In summary the majority of studies have proven the diversification effect caused by adding cryptocurrencies into investment portfolio and significantly higher risks of crypto portfolios if compared with traditional ones.

One of the core points in researches are devoted to different methods and algorithms to obtain optimal portfolio structure. Ma, Yechi & Ahmad, Ferhana & Liu, Miao & Wang, Zilong, (2020), Holovatiuk, O., (2020), Brauneis, A. & Mestel, R., (2019) and many others applied Markowitz model. Markowitz model is usually represented as quadratic programming problem and is solved accordingly.

Ankang, Li (2023), Mahboubeh Shadabfar, Longsheng Cheng (2020), Yiqian Wang, Nan Yang Yang, Qianwei Zhao (2022), Zihao Chen (2022) provided solutions based on Monte Carlo Simulation. It is a computational technique used to model the behavior of complex systems through repeated random sampling. The approach enables risk assessment and decision-making under uncertainty and also allows for the exploration of various scenarios and what-if analyses. From the other side It can be computationally demanding, especially for models with many input variables or requiring a large number of simulations. Also its results are heavily influenced by the quality of input data and assumptions.

It should be pointed out, that portfolio optimization is often applied to already predefined set of crypto assets. Very often crypto assets are chosen according to only two criteria - market capitalization value and predefined low correlations with other traditional portfolio constituents. Authors often suggest to apply different clustering procedures to group initial set of cryptocurrencies onto homogenous groups, choose the group with currently suitable risk characteristics and then add either all group participants or some of it representatives to the investment portfolio. Such decision algorithm doesn't fully take into account relations between crypto participants and other portfolio members. Thus, algorithms for choosing the most suitable assets to be added to the portfolio hasn't been researched fully yet.

The paper aim is to study effects of cryptocurrencies inclusion into an investment portfolio. To cope with the mentioned aim, the following tasks are to be completed:

- Build traditional investment portfolios which include stocks, bonds, currencies and commodities
- Build portfolios containing only cryptocurrency assets
- Build portfolios that include both traditional components and crypto assets
- Conduct a comparative assessment of obtained portfolios

Data and Methodology

It was decided to study the problem in the context of the additional factor of the COVID19 pandemic, which is still relevant today. Thus, raw data series for the current research are the daily prices for time period May, 2020 – August, 2023. The starting point of the selected time interval is the approximate time when at least some of the macroeconomic indicators participating in the study and demonstrating a significant drawdown during the initial period of the pandemic reached their pre-pandemic levels or significantly approached the pre-pandemic level.

All raw data series were derived from Barchart.com.

The initial dataset consists of two subsets. The first one represents so-cold traditional assets such as stocks, bonds, commodities and currencies and includes the following: SP500 futures(ES), gold futures (GC), USA 10 year notes futures (ZN), dollar index (DXY), commodity index (GNX).

The second one represents ten cryptocurrencies and is formed according to the highest value of market capitalization as for the end of October, 2023: Bitcoin(BTCUSD), Ethereum(ETHUSD), Tether(USDTUSD), Binance Coin(BNBUSD), XRP(XRPUSD), Cardano(ADAUSD), Dogecoin(DOGEUSD), Solana(SOLUSD), Tronix(TRXUSD), Litecoin(LTCUSD).

To reach the aim of the paper the following algorithm of investigating modern opportunities on cryptocurrency market is proposed:

Step 1. Preliminary analysis of the relationships and interdependencies between traditional assets and crypto assets.

Assets relationships are investigated through applying commonly known similarity measures such as correlation coefficients. The aim of the current step is to study correlations between crypto assets and those constituents of the traditional portfolio in order to check the possibilities for portfolio diversification.

Each asset $A_i \in A$ is characterized with the appropriate time series: $rp_{i1}, rp_{i2}, \dots, rp_{iT}$, $rp_{i,k}$ – raw price for the i-th asset at the k-th time point, $k = [1, T]$. Then correlation coefficient is calculated as follows:

$$cor_{ij} = \frac{\sum_{k=1}^{T}(rp_{ik} - \overline{rp_i})(rp_{jk} - \overline{rp_j})}{\sqrt{\sum_{k=1}^{T}(rp_{ik} - \overline{rp_i})^2 \sum_{k=1}^{T}(rp_{jk} - \overline{rp_j})^2}}$$

It is desirable to choose such initial candidates on the role of additional members into portfolio that have low correlations with its current constituents. As a result of the step the initial set A is diminished $A' \subset A$.

Step 2. Formation of the initial set of crypto assets which can be potentially included into portfolio.

Each asset $A_i \in A'$ is presented as a point in two-dimentional risk-return space:

$$A_i = (r_i, d_i),$$

r_i, d_i – the i-th asset risk and return measures.

Return and risk measures are calculated as follows:

$$d_i = \frac{1}{n}\sum_{k=1}^{n} p_{ik},\ r_i = \sqrt{\frac{1}{n}\sum_{k=1}^{n}(d_i - p_{ik})^2},\ p_{i,k} = \frac{rp_{i,k} - rp_{i,k-1}}{rp_{i,k-1}},$$

$p_{i,k}$ – rate of price change for the i-th asset at the k-th time point.

It is suggested in the current research to apply the Euclidean distance metric to assess the similarity of crypto assets in the risk-return space.

Euclidean distance between asset A_i and asset A_j is denoted as

$$E(A_i, A_j) = \sqrt{(r_i - r_j)^2 + (d_i - d_j)^2}.$$

We will apply and compare the results of the following variants of choosing crypto asset to add into the traditional portfolio:

Variant 1. The nearest to the traditional portfolio

1.1. Calculate Euclidean distances $E(AC_i, AT_j)$ between each cryptoasset (AC) and each traditional asset (AT).

1.2. Choose crypto asset with the smallest Euclidean distance

$$AC^* = arg\min_{i,j} E(AC_i, AT_j)$$

Variant 2. The furthest to the traditional portfolio

1. Calculate Euclidean distances $E(AC_i, AT_j)$ between each cryptoasset (AC) and each traditional asset (AT).
2. Choose crypto asset with the biggest Euclidean distance

$$AC^* = arg\max_{i,j} E\left(AC_i, AT_j\right)$$

Variant 3. The nearest to the traditional portfolio centroid:

1. Calculate centroid coordinates for traditional assets subset:
2.

$$C = (r_c, d_c),$$
$$r_c = \frac{1}{n}\sum_{k=1}^{n} r_k, \; d_c = \frac{1}{n}\sum_{k=1}^{n} d_k,$$

r_k, d_k – risk and return values for the k-th traditional asset,
n – number of traditional assets.

3. Calculate Euclidean distances between each crypto asset (AC) and cetroid (C):

$$E(AC_i, C) = (r_i - r_c)^2 + (d_i - d_c)^2$$

4. Choose crypto asset with the smallest Euclidean distance $E(AC_i, C)$

$$AC^* = arg\min_{i} E(AC_i, C)$$

Variant 4. The furthest to the traditional portfolio centroid:

$$AC^* = arg\max_{i} E(AC_i, C)$$

Step 3. Obtain efficient frontier for portfolios with different initial composition of assets.

Portfolio return is determined as the weighted average sum of the returns of individual assets:

$$D_p = \sum_i w_i d_i$$

w_i - weight of the i-th asset.

Portfolio risk:

$$R_p^2 = \sum_i w_i^2 r_i^2 + \sum_i \sum_{j, j \neq i} w_i w_j r_i r_j \, \rho_{ij}$$

ρ_{ij} - the correlation coefficient between the returns on asset i and j.

Thus, every given portfolio can be plotted as a point in risk-return space. According to Akhilesh Ganti (2023) the set of such points which are portfolios that offer the highest expected return for a defined level of risk or the lowest risk for a given level of expected return is called efficient frontier. Portfolios that lie below the efficient frontier are sub-optimal because they do not provide enough return for the given level of risk. Portfolios that cluster to the right of the efficient frontier are sub-optimal because they have a higher level of risk for the defined rate of return.

The optimization model to find optimal portfolio structure (optimal asset weights) is presented in the following two general forms (see Markowitz, H. (1952), Vollmer, M. (2015), Sharpe, William F. (1964)).

Form 1. Minimize portfolio risk given the prespecified portfolio return:

$$\min_{w_i} R_p^2$$

subject to

$$D_p \geq \varphi$$
$$\sum_i w_i = 1$$
$$w_i \geq 0$$

Form 2. Maximize portfolio return given the prespecified portfolio risk:

$$\max_{w_i} D_p$$

subject to

$$R_p^2 \leq \theta$$
$$\sum_i w_i = 1$$
$$w_i \geq 0$$

Here Markowitz model (Form 1) is represented as Quadratic Programming problem (the objective function is quadratic, and the constraints are linear). It can be solved using various optimization algorithms, such as interior-point methods, active-set methods, or sequential quadratic programming methods.

The current model can be also performed as a convex optimization problem. In this case it can be solved using various convex optimization algorithms (interior-point methods, gradient descent, or alternating direction method of multipliers).

Monte Carlo simulation or some heuristic algorithms also can be implemented for the case. In the former case random portfolios have to be generated based on different asset weight combinations. For each generated portfolio, the expected return and risk are computed. By simulating a large number of portfolios, one can approximate the efficient frontier and identify the optimal portfolio allocation. In the latter case genetic or evolutionary algorithms can be applied to solve the model.

Step 4. Comparative assessment and result analysis

To effectively compare different portfolios, it is proposed to apply Sharp ratio – the standard reward-to-risk metric.

The portfolio reward is measured as the portfolio's excess return, which is equal to the difference between the portfolio's return and the return on a "risk-free" investment:

$$R_{sharp} = \frac{D_p - D_f}{R_p}$$

R_p – portfolio risk,
D_p – portfolio return,
D_f - risk-free rate of return, typically representing the return on a risk-free asset such as government bonds.

In the current research we will ignore risk-free rate because our goal is just to compare different portfolios and not to obtain the absolute risk-return measure.

The reward-to-risk metric is a measure used to assess the performance of an investment relative to the amount of risk taken. It evaluates how much return an investment generates for each unit of risk assumed. The portfolio reward is measured as the portfolio's return; the measure of portfolio risk is the standard deviation of the portfolio returns. Some other reward-to-risk metrics besides the Sharpe ratio such as Sortino ratio, Treynor ratio, Calmar ratio may be additionally calculated(see Sharpe, William F. (1966), Tobin, J. (1969)). If two portfolios have the same return but a different reward-to-risk ratio, the portfolio with the higher reward-to-risk ratio can be chosen if all other characteristics are applicable.

Results and Discussion

Firstly, the preliminary correlation analysis was initialized. The results can be seen at Table 2.

Table 2. Correlations matrix (Calculated by the authors)

	BTCUSD	ETHUSD	USDTUSD	BNBUSD	XRPUSD	ADAUSD	DOGEUSD	SOLUSD	TRXUSD	LTCUSD
GC	-0,060	0,010	0,165	-0,071	-0,040	-0,200	-0,077	-0,092	-0,038	-0,072
GNX	0,277	0,493	-0,362	0,607	0,216	0,219	0,276	0,371	0,544	0,007
ZN	0,126	-0,102	0,358	-0,308	0,109	0,236	0,064	0,037	-0,333	0,301
ES	0,719	0,866	-0,037	0,820	0,672	0,637	0,613	0,662	0,770	0,517
DXY	-0,423	-0,152	-0,392	0,067	-0,370	-0,419	-0,292	-0,109	0,047	-0,590

The pair correlations for cryptocurrencies subset is not demonstrated here, because it is well known that the majority of crypto assets are highly correlated so the minor differences in correlation levels between them are not the topic of interest here. Our goal is to research correlations between crypto assets and those constituents of the traditional portfolio. This research aims to answer the questions about diversification opportunities of cryptocurrencies when adding them into traditional portfolio.

According to the Table 2, one can observe rather strong positive correlation between SP500 index and all cryptocurrencies except USDT. The most correlation values exceed 0.6 level (except correlation between SP500 and LTC). The correlation of the remaining traditional assets and cryptocurrencies, with rare exceptions, is quite weak. This allows to formulate and investigate hypothesis about the possibility of applying cryptocurrencies in a portfolio to increase the diversification effect.

The implementation results of the described above criteria are presented at the Table 3 and on the Fig. 3. According to the Table 3, the nearest distance is detected between BTC and GNX, so BTC is taken according to the 1-st criterion. The largest distance value is determined for the pair SOL-ZN, so SOL is taken according to the 2-nd criterion.

Table 3. Euclidian distances (Calculated by the authors)

	GC	GNX	ZN	ES	DXY
BTCUSD	0,00271	0,00199	0,00306	0,00236	0,00267
ETHUSD	0,00469	0,00396	0,00505	0,00434	0,00466
BNBUSD	0,00610	0,00536	0,00646	0,00574	0,00607
XRPUSD	0,00726	0,00670	0,00754	0,00701	0,00727
ADAUSD	0,00538	0,00473	0,00570	0,00507	0,00536
SOLUSD	0,00980	0,00915	0,01011	0,00949	0,00978
TRXUSD	0,00446	0,00380	0,00479	0,00414	0,00444
DOTUSD	0,00497	0,00449	0,00522	0,00475	0,00499

Figure 3 presents distances between crypto assets and the centroid calculated for traditional set. It can be seen that ADA is the nearest asset and SOL is the furthest one. It also should be noted that such cryptocurrencies as DOT, TRX, ETH and BNB are situated rather compact and have distances that do not differ significantly from the ADA's distance. In fact, they may be also taken as proxy according to the criterion of the nearest distance from the centroid. Thus, ADA was chosen according to the third criterion, SOL was chosen according to the fourth criterion.

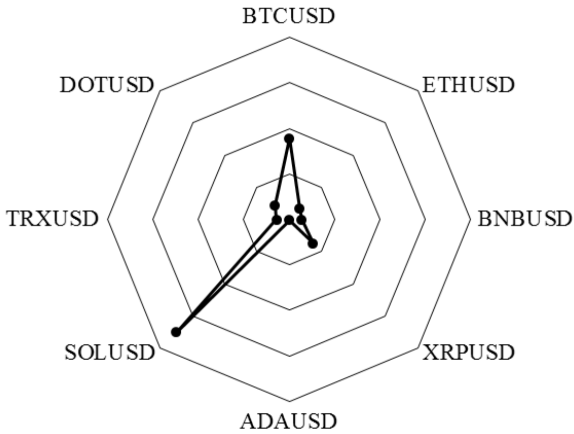


Fig. 3. Distances from the centroid of traditional assets (Calculated by the authors)

Table 4 presents optimal portfolios which were obtained as the result of Markowitz model implementation. It has three portfolios which were constructed as the result of Step 2 implementation (they are named as BTC, SOL and ADA), portfolio with only traditional constituents (TRD), portfolio with only crypto assets (ONLY_CRYPTO) and portfolio with all types of assets (TRD_CRYPTO). Model inputs included the whole set of assets mentioned earlier but not all of them are present in the structure of optimal portfolios.

Table 4. Portfolios (Calculated by the authors)

Portfolio name	Model Constituents
TRD	GC, GNX, ZN, ES
BTC	GC, GNX, ZN, ES, BTCUSD
SOL	GC, GNX, ZN, ES, SOLUSD
ADA	GC, GNX, ZN, ES, ADAUSD
ONLY_CRYPTO	BTCUSD, ETHUSD, USDTUSD, BNBUSD, XRPUSD, ADAUSD, SOLUSD, TRXUSD
TRD_CRYPTO	GC, GNX, ZN, ES, BTCUSD, ETHUSD, USDTUSD, BNBUSD, XRPUSD, ADAUSD, SOLUSD, TRXUSD

Let's discuss obtained portfolios in detail. Efficient frontier was constructed for each portfolio and ten alternatives of portfolio structure were determined. Among these ten alternatives the first one has the minimum risk and return values, the tenth is characterized by the largest risk and return values. Sharpe Ratio was calculated ten times. Also the eleventh alternative of portfolio structure with the largest Sharpe Ratio was determined.

Fig. 4 presents the results of traditional portfolio modeling.

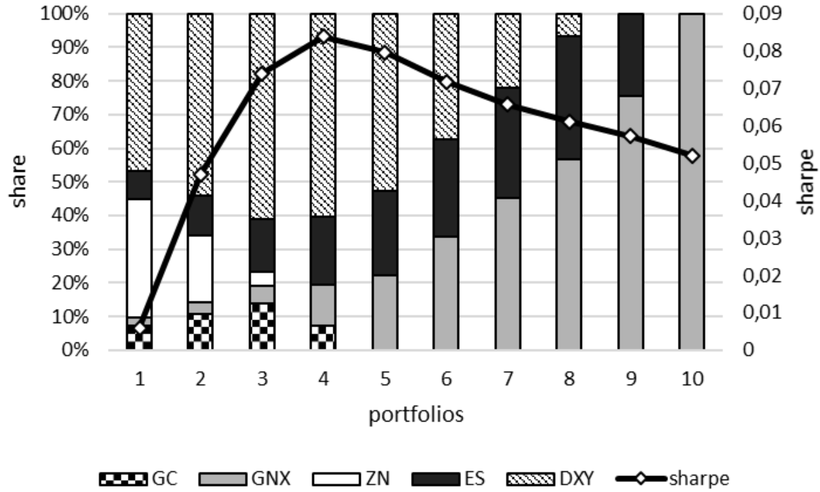


Fig. 4. Efficient frontier: TRD portfolio (Calculated by the authors)

Among ten presented alternatives the largest Sharpe Ratio is achieved by the combination of gold, other commodities, stocks and currency; the most significant share is occupied by currency (nearly 60%). Bonds are present in only first three alternatives, the greater the share of bonds, the lower the Sharpe ratio for the model results. Gold is present in only first four alternatives and its share doesn't exceed 14% in any of them. The highest Sharpe Ratio values achieved with half or more costs allocated into currency (see portfolio 3, 4 and 5).

Next let's discuss ONLY_CRYPTO portfolio – it consists only of crypto assets. Initially all crypto assets mentioned on Fig. 2 were included as model inputs but only six of them became constituents of efficient frontier portfolios: ETHUSD, USDTUSD, BNBUSD, XRPUSD, SOLUSD, TRXUSD. Such assets as DOGEUSD, BTCUSD and DOTUSD are not present at the final results (see Fig. 5).

It is interesting to note that perhaps DOGEUSD may have been excluded as the riskiest one. BTCUSD and DOTUSD can be classified as its opposites – both of them have rather low risk values (BTCUSD has the second risk rank after USDTUSD).

Here you can see to opposite one-element portfolios: the first portfolio consists only of USDTUSD and is characterized with the lowest Sharpe Ratio; the tenth portfolio consists only of SOLUSD. In general, here the majority of portfolios (from the second to the ninth) have very similar values of Sharpe Ratio, so to make the final decision investor can apply some additional criteria.

As for the portfolio structure it can be seen that ETHUSD, XRPUSD and TRXUSD are present in the majority of portfolios but their shares are rather low: the non-zero share of TRXUSD fluctuates from to 1.2% to 8.6%; the appropriate values for ETHUSD are 1.4% and 10.6% respectively; the appropriate values for XRPUSD are the lowest (0.2% and 1.4%

respectively). The core constituents according to the Fig. 5 are USDTUSD (1.2% - 85.9%), BNBUSD (7% - 48.9%) and SOLUSD (4.2% - 53.5%).

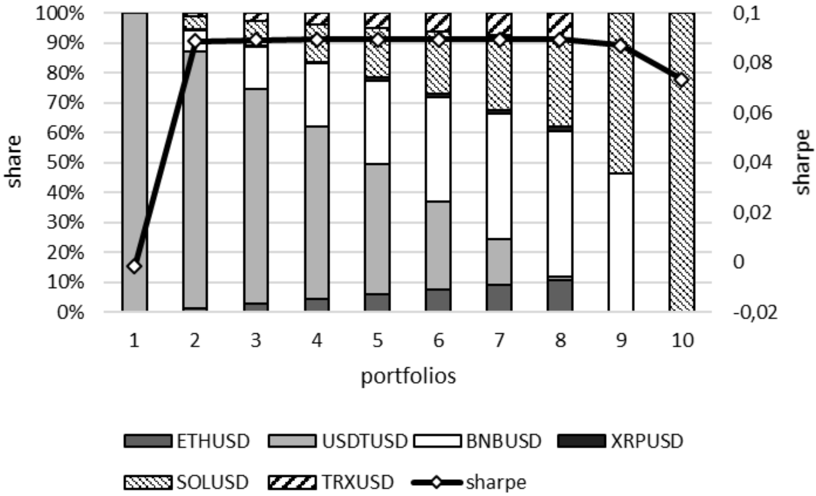


Fig. 5. Efficient frontier: ONLY_CRYPTO portfolio (Calculated by the authors)

Fig. 6 presents efficient frontier for TRD_CRYPTO portfolio. Here all crypto assets and all traditional assets presented at Fig. 2 were applied as initial inputs for the model. As a result, eleven constituents are present in different proportions for ten optimal efficient frontier portfolios. All traditional assets can be seen at Fig. 6. The smallest share values and the rarest presence are observed for gold (GC), stocks (ES) and bonds (ZN). Stocks are only present in the second portfolio with the respective share of 9.1%; gold assets are present in the fist and in the second portfolios and occupy 0.1% and 1.8% of total portfolio value respectively; bonds are included into the first portfolio and occupy just 0.3%. thus the core traditional constituents for the majority of portfolios are currency (DXY) and commodities (GNX).

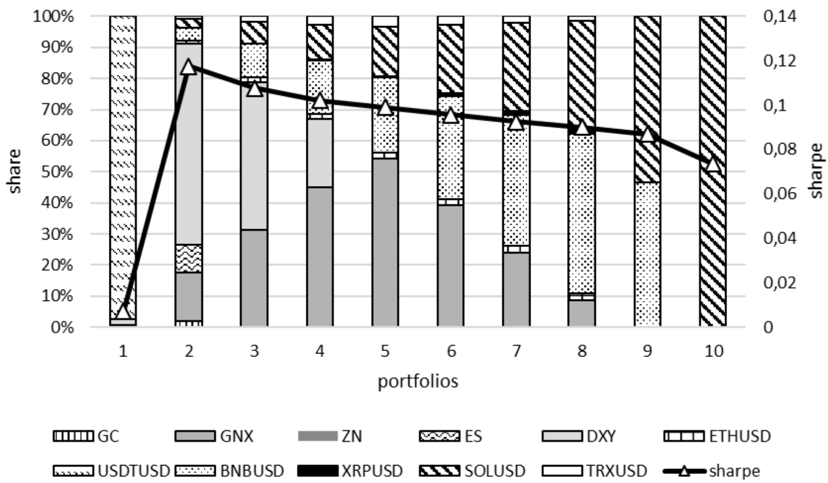


Fig. 6. Efficient frontier: TRD_CRYPTO portfolio (Calculated by the authors)

Let's discuss crypto participants. Only six crypto assets from the initial set are present at Fig. 6.

USDTUSD was included only in the first portfolio with the share of 97.3%. It is the portfolio with the lowest Sharpe Ratio and in fact it just replicates the corresponding dominant asset.

ETHUSD, XRPUSD and TRXUSD are present at different portfolios but their shares are rather small. The core crypto participants are BNBUSD and SOLUSD. BNBUSD is present in eight portfolios and occupies share from 4.3% to 51.3%. SOLUSD present in nine portfolios and occupies share from 2.7% to 100%.

The second portfolio has the highest Sharpe Ratio and 91.4% of its assets are traditional ones. The total share of crypto assets increases from the third portfolio to the eighth one and is accompanied by a simultaneous decrease in the Sharpe ratio. 9-th and 10-th portfolios are totally constructed of crypto assets.

The next three portfolios (BTC portfolio, ADA portfolio and SOL portfolio) were constructed as follows: initial inputs for the model were all traditional assets plus the only previously defined crypto asset.

Fig. 7 presents efficient frontier obtained for BTC portfolio.

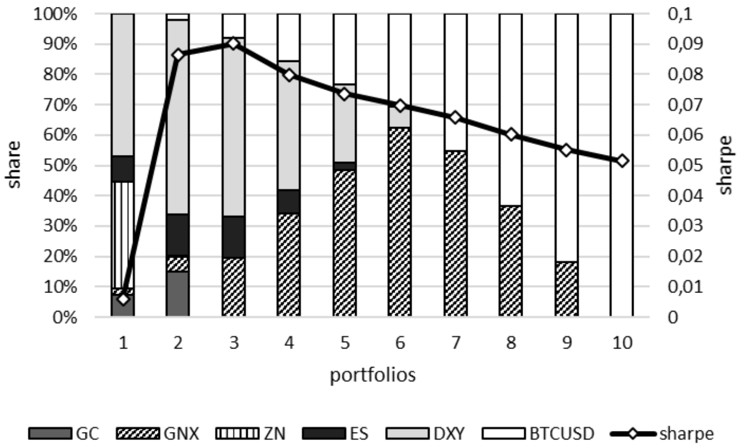


Fig. 7. Efficient frontier: BTC portfolio (Calculated by the authors)

The first portfolio has the lowest Sharpe Ratio and consists only of traditional assets. The tenth portfolio has the only member which is the corresponding crypto currency. In general, the greater The Sharpe Ratio the less is the share of BTCUSD in the portfolio structure. All traditional assets are present at obtained portfolios but their shares vary significantly. GNX is the most common asset, DXY is present in the first six portfolios only. Their shares fluctuate in the following ranges: [2.4%; 48.5%] for GNX and [6.9%;64.1%] for DXY. ES can be observed just in the first five portfolios but its maximum share is just 13.8% (portfolio 2). ZN is present with rather valuable share of 35.2% only in the second portfolio.

Fig. 8 presents efficient frontier obtained for ADA portfolio.

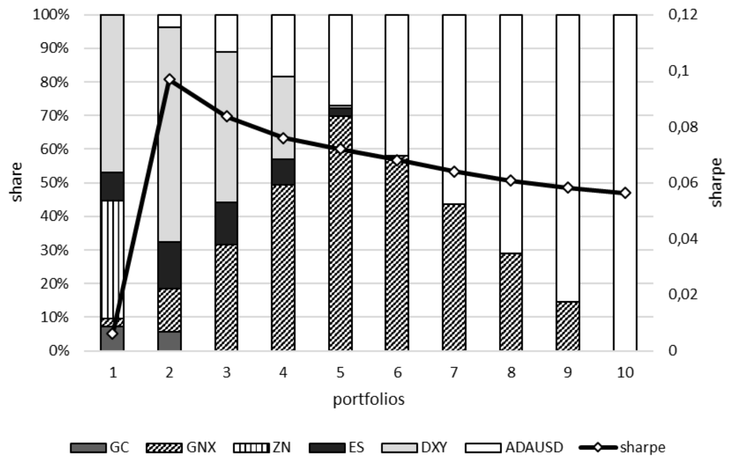


Fig. 8. Efficient frontier: ADA portfolio (Calculated by the authors)

Here traditional assets are mostly presented by commodities and currency. Gold as separate asset is present only in the first two portfolios and each time its share doesn't exceed 10%. Bonds are included only in the first portfolio which doesn't have crypto part in the structure. The second portfolio is characterized by the highest Sharpe Ratio and consists of all assets except bonds. Starting from the sixth portfolio the structure consists only of two members in different proportions – crypto asset and commodities.

Fig. 9 presents efficient frontier obtained for SOL portfolio.

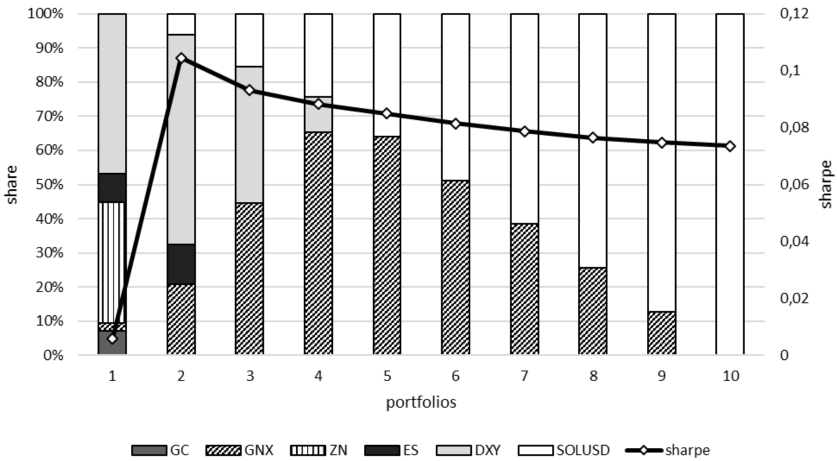
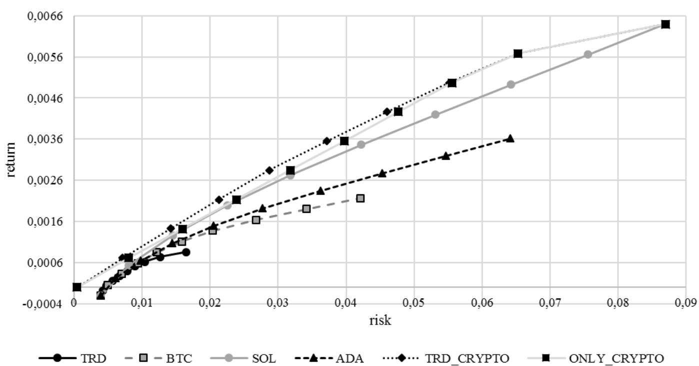


Fig. 9. Efficient frontier: SOL portfolio (Calculated by the authors)

The situation for SOL portfolio is rather similar with those for previous ADA portfolio. One of the dissimilarities can be observed for the portfolio with the highest Sharpe Ratio. Here there are only four members in its structure (crypto asset, commodities, stocks and currency). The second point is that stocks are present only in the first two portfolios.

Fig. 10 shows efficient frontiers for all obtained portfolios.



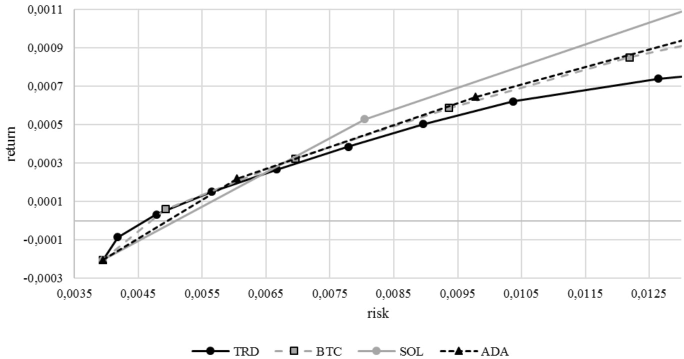


Fig.10. Efficient frontiers (Calculated by the authors)

Let's discuss major peculiarities of the obtained results.

Firstly, it should be noted that there are portfolios which are situated in the area of negative returns. This is a consequence of the inclusion of bonds (showed negative returns during the considered time interval). Because bonds' part in the portfolios with the lowest risk was rather significant (e.g. in the traditional portfolio case the first two portfolios on the efficient frontier have 86% and 72% of bonds respectively) it effected the total portfolio return. It should be noted that in the zone of negative returns the traditional portfolio efficient frontier is higher than those ones for all other portfolios. It means that the traditional portfolio provides less risk with the same return in comparison with other portfolios.

Secondly, as expected, the inclusion of crypto assets in the portfolio led to a significant expansion of the range of risk and return for portfolios that lie on the efficient frontier.

Table 5 presents final characteristics of all researched portfolios: the highest Sharpe Ratio on the given efficient frontier, risk and return of the portfolio with the highest Sharpe Ratio, the share of crypto assets in the given portfolio and the growth rate of the Sharpe Ratio relative to those one for the traditional portfolio.

As it can be seen, the highest Sharpe Ratio growth rate corresponds to the solution when the portfolio includes eight crypto assets. However, this option will most likely remain only as a purely theoretical example, since it will require a significant increase in portfolio management costs compared to other options. This portfolio is also characterized by the maximum share of crypto assets (32%) and the maximum level of risk.

Table 5. Portfolio characteristics (Calculated by the authors)

Portfolio name	Risk	Return	Sharpe	Crypto Percent	Sharpe ratio growth rate, %
TRD	0.010996	0.000666	0.060546	0	0
BTC	0.014095	0.000997	0.07071	0.188507	16.7874

ADA	0.015615	0.001158	0.074139	0.150068	22.451
SOL	0.019273	0.001709	0.088668	0.169347	46.4466
ONLY_CRYPTO	0.056223	0.005031	0.089491	1	47.807
TRD_CRYPTO	0.021939	0.002199	0.100247	0.322174	65.5709

From the point of view of practical implementation, portfolio variants with one crypto asset look much more realistic. According to the Table 5, the mentioned portfolios demonstrate approximately equivalent Sharpe Ratio levels in the interval (0.07-0,09), and the share of crypto asset – in interval (15%-19%). BTC portfolio shows the highest share of cryptocurrency in the portfolio and the lowest Sharpe ratio. The situation with SOL is exactly the opposite – this portfolio has the lowest share of the cryptocurrency and the highest Sharpe ratio.

It should be noted that for all researched portfolio structure variants the option that corresponds to the highest Sharpe Ratio does not include gold and bonds. The obtained portfolio structures can be seen at the Fig. 11. The share of GNX is approximately equal or exceeds 50% for all portfolios. For BTC, ADA and SOL the ratio of the share of stocks and the share of crypto asset fluctuates between 1.32 and 1.98. For TRD_CRYPTO portfolio the opposite situation is observed - the total share of crypto assets is much larger than the share of stocks.

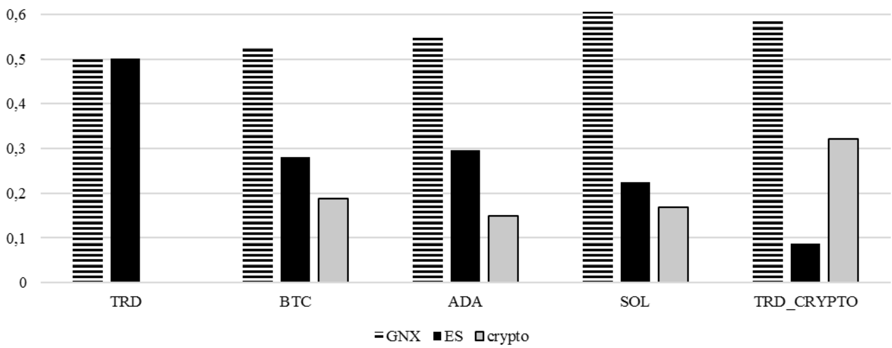


Fig. 11. Structure of portfolios with the highest Sharpe Ratio (Calculated by the authors)

Fig. 12 presents dynamics of TRD portfolio, BTC portfolio, ADA portfolio and SOL portfolio, which are measured as if initial value of = $ 10 000 is invested in proportions according to modeling results for the portfolio with maximum Sharpe Ratio value. Total portfolio volume at the t-th period is assessed as follows:

$$V_t = \sum_{i=1}^{n} m_i p_{it}, m_i = \frac{w_i I}{p_{io}},$$

m_i –i-th asset number of units invested,
w_i - i-th asset share in the portfolio,
I – total invested amount,
p_{io} –i-th asset price at the beginning of investment period.

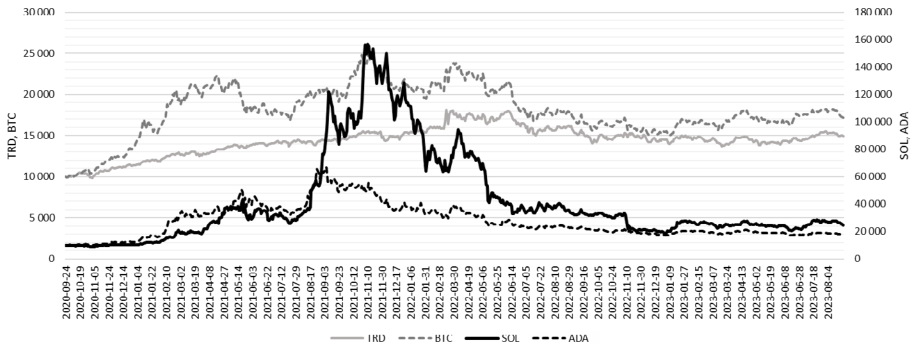


Fig. 12. Portfolios dynamics (Calculated by the authors)

Results are shown in two different scales: left-hand scale is for TRD and BTC, right-hand scale is for SOL and ADA. The reasons for such scaling are significant fluctuations in the latter portfolios estimated financial results: both of them have skyrocketed and then have dropped different times. Fig. 13 presents drawdowns to show these facts more accurate. Drawdowns were calculated as follows:

$$DD_t = \frac{(V_t - \max_{k\in[1,t]} V_k)}{\max_{k\in[1,t]} V_k} \cdot 100\%$$

Fig. 13 shows box and whisker plots which allow to assess significant differences in drawdown ranges for researched portfolios. According to the provided drawdown formula the minimum value is always zero, maximum values vary and it can be seen that they exceed 50% for SOL and ADA portfolios. These plots also help to analyze interquartile ranges. Here for TRD and BTC their values are approximately 18% and 22%, from the other hand, values for ADA and SOL are nearly 50% and 70%, respectively.

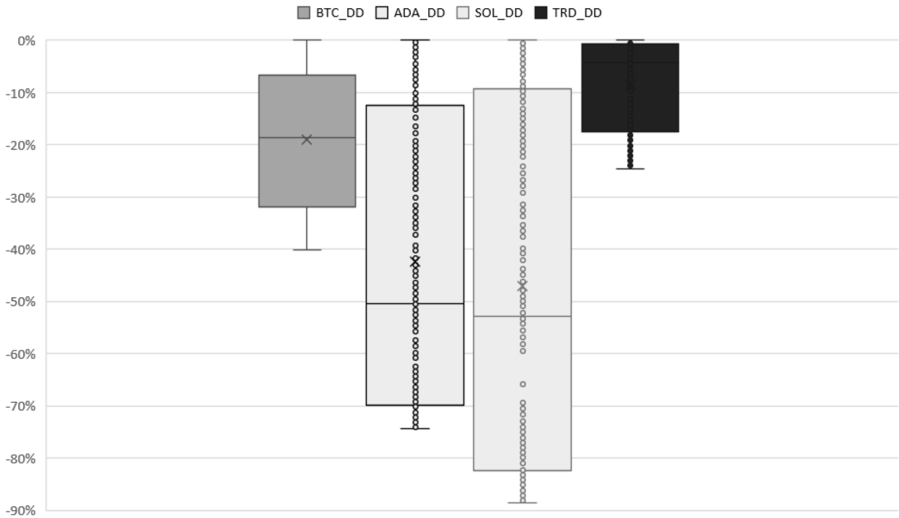


Fig. 13. Box and whisker (Calculated by the authors)

At the end of period the annual gain values are the follows: 26% for TRD, 37% for BTC, 41% for ADA, 77% for SOL. Thus, SOL portfolio can be chosen as a proxy because of its maximum gain but for risk averse investors BTC portfolio may be more preferable. ADA has produced just 4% of additional gain if to be compared with BTC that is why currently it should be recommended.

It should be noted that such numbers were obtained as the result of implementation of the simple buy and hold investment strategy and no procedures were held to rebalance portfolios during the researched period. perhaps rebalancing would cause some differences in resulting risk and return values but not change core conclusions in general.

Conclusions and directions for future research.

In recent time the situation with cryptocurrency market liquidity has improved significantly, that is why the more institutional and private investors are demonstrating intentions to deal with this kind of assets. The results obtained in the current research allow to conclude that there are opportunities of portfolio efficiency increase via crypto assets inclusion in its structure. It was distinguished that the inclusion of crypto asset caused rather significant growth in the portfolio risk-return measures. The mentioned increase value varies noticeably and depends on the particular kind of crypto assets, their total share and total number of different crypto assets. Theoretically, more crypto assets included into portfolio cause the larger growth of efficiency measure but from the practical point of view additional transaction costs will probably worsen the final results of portfolio performance thus currently we recommend to apply only one crypto asset as additional constituent of the traditional portfolio.

Further research will be devoted to investigation of additional criteria added to the basic Markowitz model. There are only two types of so cold traditional asset classes in the resulting modeling results: commodities and stocks. It may be useful to implement modeling with additional restriction on the minimum acceptable share value for each asset class. For instance, it may be potentially promising to include assets of fixed income class.

Moreover, currently each asset class is presented by the appropriate market index, so it may be promising to study assets within a class and provide a procedure to choose the most applicable participants from the point of their interdependencies with crypto market.

Another issue which can be fixed is the abandon of short sales (in the model it means that currently asset weights can't assume negative values). Nowadays the variety of traditional market traded instruments are allowed for short sales and it can be implemented for crypto assets too.

References

1. Akhilesh Ganti (June 28, 2023). *Efficient Frontier: What It Is and How Investors Use It.* https://www.investopedia.com/terms/e/efficientfrontier.asp
2. Ankang, Li (2023). Portfolio optimization by Monte Carlo simulation. *Advances in economics management and political sciences, 50*(1), 133-138. https://doi.org/10.54254/2754-1169/50/20230568
3. Ankenbrand, T., & Bieri, D. (2018). Assessment of cryptocurrencies as an asset class by their characteristics. *Investment Management and Financial Innovations, 15*(3). https://doi.org/10.21511/imfi.15(3).2018.14
4. Asanga Jayawardhana & Sisira R N Colombage (2023). Portfolio diversification possibilities of cryptocurrency: global evidence. *Applied Economics.* https://doi.org/10.1080/00036846.2023.2257928
5. Barchart. (n.d.). ETF Finder. Retrived April 11, 2024, from https://www.barchart.com/etfs-funds/etfs-by-asset-class?dropdown1=352&viewName=main&dropdown2=462
6. Brauneis, A., Mestel, R. (2019). Cryptocurrency-portfolios in a mean-variance framework. *Finance Research Letters, 28*(C), 259-264. https://doi.org/10.1016/j.frl.2018.05.008
7. Chen, Tian (2021). Looking for the Optimal Cryptocurrency Portfolio: Benefits of Diversification. *6th International Conference on Financial Innovation and Economic Development (ICFIED 2021).* https://doi.org/10.2991/aebmr.k.210319.101
8. Coinbase. (n.d.). Explore the cryptoeconomy. Retrieved April 11, 2024, from https://www.coinbase.com/explore
9. CoinMarketCap. (n.d.). Binance. Retrived April 11, 2024, from https://coinmarketcap.com/exchanges/binance/
10. Colombo, J.,Cruz, F., Paese, L., Cortes, R. (2021). The Diversification Benefits of Cryptocurrencies in Multi-Asset Portfolios: Cross-Country Evidence. http://dx.doi.org/10.2139/ssrn.3776260

11. Corbet, S., Lucey, B., Urquhart, A., Yarovaya, L. (2019). Cryptocurrencies as a financial asset: A systematic analysis. *International Review of Financial Analysis, 62*(C), 182-199. https://doi.org/10.1016/j.irfa.2018.09.003
12. Damianov, D. S., Elsayed, A. H. (2020). Does Bitcoin add value to global industry portfolios? *Economics Letters, 191*(C). https://doi.org/10.1016/j.econlet.2019.108935
13. Elendner, H., Trimborn, S., Ong, B., Teik Ming Lee. (2016). The Cross-Section of Crypto-Currencies as Financial Assets: An Overview. *SFB 649 Discussion Papers SFB649DP2016-038, Sonderforschungsbereich 649, Humboldt University, Berlin, Germany.* https://ideas.repec.org/p/hum/wpaper/sfb649dp2016-038.html
14. Holovatiuk, O. (2020). Cryptocurrencies as an asset class in portfolio optimisation. *Central European Economic Journal, 7*(54), 33-55. https://doi.org/10.2478/ceej-2020-0004
15. IGWT Report (n.d.). Chart Market Capitalization of Gold and Bitcoin. Retrived April 11, 2024, from https://ingoldwetrust.report/chart-gold-Bitcoin-marketcap/?lang=en
16. Jiaqi Qin, Shansong Huang, Boying Yang, Yilin Ma, Zheng Tao, Shuqi Chen (2022). *Analyze the Impact of Bitcoin on Stock Portfolio's Risk and Return Based on Past 3 Years' Data.* https://doi.org/10.2991/978-94-6463-010-7_121
17. Kajtazi, A., Moro, A. (2019). The role of Bitcoin in well diversified portfolios: A comparative global study. *International Review of Financial Analysis, 61*(C), 143-157. https://doi.org/10.1016/j.irfa.2018.10.003
18. Lorenzo, L., Arroyo, J. (2023). Online risk-based portfolio allocation on subsets of crypto assets applying a prototype-based clustering algorithm. *Financial Innovation, 9*(1). https://doi.org/10.1186/s40854-022-00438-2
19. Ma, Yechi & Ahmad, Ferhana & Liu, Miao & Wang, Zilong (2020). Portfolio optimization in the era of digital financialization using cryptocurrencies. *Technological forecasting and social change, 161*(C). https://doi.org/10.1016/j.techfore.2020.120265
20. Maghsoodi, Abtin Ijadi (2023). Cryptocurrency portfolio allocation using a novel hybrid and predictive big data decision support system. *Omega, 115.* https://doi.org/10.1016/j.omega.2022.102787
21. Mahboubeh Shadabfar & Longsheng Cheng (2020). Probabilistic approach for optimal portfolio selection using a hybrid Monte Carlo simulation and Markowitz model. *Alexandria Engineering Journal, 59*(5), 3381-3393. https://doi.org/10.1016/j.aej.2020.05.006
22. Markowitz, H. (1952). Portfolio selection. *Journal of Finance, 7*(1), 77-91.
23. Sharpe, William F. (1964). Capital asset prices: a theory of market equilibrium under conditions of risk. *The Journal of Finance, 19*(3), 425–442.
24. Sharpe, William F. (1966). Mutual Fund Performance. in Part 2: Supplement on Security Prices. *Journal of Business, 39*(1), 119–138.
25. Shaun Paul Lee (2023, May 30). *How Big is the Crypto Derivatives Market?* https://www.coingecko.com/research/publications/crypto-derivatives-market#:~:text=How%20Big%20is%20the%20Crypto,trading%20volume%20of%20%242.95%20trillion
26. Sifma (July 24, 2023). Capital Markets Fact Book. https://www.sifma.org/resources/research/fact-book/
27. Tobin, J. (1969). A general equilibrium approach to monetary theory. *Journal of Money, Credit and Banking, 1*(1), 15–29.

28. Vollmer, M. (2015). *Beta-return Efficient Portfolio Optimisation Following the CAPM*. Wiesbaden, Germany: Springer Gabler.
29. Yiqian Wang, Nan Yang Yang & Qianwei Zhao (2022). Application of Monte Carlo simulation in calculating the maximum sharpe ratio based on American funds. *Proceedings of the 4th International Conference on Economic Management and Model Engineering*. https://doi.org/10.4108/eai.18-11-2022.2327143
30. Zihao Chen (2022). Asset allocation strategy with Monte Carlo simulation for forecasting stock price by ARIMA model. *Proceedings of the 2022 13th International Conference on E-Education, E-Business, E-Management, and E-Learning*, 481–485. https://doi.org/10.1145/3514262.3514331