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MODELING OF PRICING IN ENVIRONMENTAL ECONOMICS

Andrii VERSTIAK
Chernivtsi National University, Ukraine, Department of Economics
Kotsubinsky Str., 2, 58012, Chernivtsi, Ukraine
Phone: +38062655588
E-mail: a.verstyak@chnu.edu.ua

Vasyl GRYGORKIV
Chernivtsi National University, Ukraine, Department of Economics
Kotsubinsky Str., 2, 58012, Chernivtsi, Ukraine
Phone: +380505281600
E-mail: emmvsh@gmail.com

Mariia GRYGORKIV
Chernivtsi National University, Ukraine, Department of Economics
Kotsubinsky Str., 2, 58012, Chernivtsi, Ukraine
Phone: +380506418085
E-mail: gmvmarichka@gmail.com

Abstract. The problem of including the ecological constituent in the price and the necessity of the models construction of pricing, taking into account the terms of environmental economics, are analyzed. The article develops the determined models of the pricing to be used for the decision making in prognostication of the price indexes in diversified environmental economics. On its basis it is possible to realize the process of pricing in the eco-economic system. These models are taken to the proper equivalents, that is why its practical introduction is based on the mathematical and computer providing of the determined tasks.

JEL Classification: C21, C67, C68.
Keywords: environmental economics, pricing, inter-sectoral balanced models, input-output, linear models of pricing, eco-economic system.
Reikšminiai žodžiai: aplinkos ekonomika, kainodara, tarpsektorinis balanso modelis, indėlis-išdirbis, linijinis kainodaros modelis, ekologinės ekonomikos sistema.

Introduction

Since the main source of pollution is production, pollution is the result of economic activity, so this result must be reflected in the models of industrial and economical systems, particularly in the linear input-output models. These linear models can more fully identify the features of pricing activity and on the basis of it to forecast changes in price indices by changing certain elements of input-output balance. In recent years, progresses have been made to address some of these challenges: on the environmental data front, efforts have been made to compile environmental data beyond criteria pollutants for input-output applications [1–4]. Multi-regional input-output analysis has been applied to better assess pollution embodied in international trades [5–7].

There is a huge amount of literature on economic development and environmental sustainability. The literature concerns problems of measuring and implementation of it in eco-economic and environmental policies. But complexity and diversity of the eco-economic systems and sustainable development require further investigation with the aim of the construction of new methods (or perfection the existing ones) of the solving socioeconomic tasks and saving natural-resources potential with comprehensive consideration of eco-economic factors in pricing. Therefore the aim of the article is economic modeling of pricing in eco-economic systems with linear inter-sectoral links.

The problem of the forecasting of the prices in ecological economy can be realized on the basis of the linear input-output model of interagency environmental and economic balance, reflecting the simultaneous operation of activities: main (the branches of material production) and secondary (the industry of destruction of pollutants). On this basis it is possible to realize the process of pricing in the environmental economics.

In this paper we consider the problem of the prognostication of balanced prices in environmental economics on the basis of Leontief-Ford model [8] that generalize Leontief's classical input-output model [9–11]. Over the last years, input-output analysis has been successfully applied to address various environmental and energy issues (Ayres and Kneese, 1969; Wright, 1974; Berry and Fels, 1973; Bullard and Herendeen, 1975; Bullard et al., 1978; Chapman, 1974; Cleveland et al., 1984; Duchin, F., 1992; Duchin and Lange, 1994) [12–20].

Leontief-Ford reflects the simultaneous operation of two kinds of activities: main (the branches of material production) and secondary (the industry of destruction of pollutants).

1. The environmental input-output model with linear inter-sectoral links

One of the variants of the direct Leontief-Ford model can be written as follows:

$$\begin{cases} x^{(1)} = A_{11}x^{(1)} + A_{12}x^{(2)} + y^{(1)} \\ x^{(2)} = A_{11}x^{(1)} + A_{12}x^{(2)} - y^{(2)}, \end{cases} \tag{1}$$

where $x^{(1)} \in \mathbb{R}_+^n$ – vector of total output of main production ($\mathbb{R}_+^n$ —positive orthant of n-dimensional vector space);

$x^{(2)} \in \mathbb{R}_+^m$ – vector of total destroyed industrial contaminant (that total output vector of support sector);

$y^{(1)} \in \mathbb{R}_{+}^{n}$ – vector of final output;

$y^{(2)} \in \mathbb{R}_{+}^{m}$ – vector of undestroyed industrial contaminant;

$A_{11} = \left(a_{ij}^{(11)} \right)_{i,j=1}^{n}$ – the square matrix of spending of the good *i* for producing the good *j* in number $x_j^{(1)}$;

$A_{12} = \left(a_{is}^{(12)} \right)_{i,s=1}^{n,m}$ – the rectangular matrix of spending of the good *i* for destroying the contaminant *s* in number $x_s^{(2)}$;

$A_{21} = \left(a_{lj}^{(21)} \right)_{l,j=1}^{m,n}$ – the rectangular matrix of production of the contaminant *l* during the production process of the good $x_j^{(1)}$ in number *j*;

$A_{22} = \left(a_{ls}^{(22)} \right)_{l,s=1}^{m}$ – the square matrix of production of the contaminant *l* during the destroying process of the contaminant *s*.

The components of above vectors and matrixes are positive because it reflects the real economic sense. The meaning of model (1) is obvious: the first equality—good's distribution of material production on the spending in the main and secondary activities and final output; the second equality—balanced interrelation that concerns pollutants and means that the amount of destroyed contaminants equals the difference between the amount of the all produced pollutants and undestroyed ones.

Similarly with the classical input-output model the formula (1) can be constructed on the base of appropriate scheme by "rows." The formula (1) is well investigated on the problem of existence of positive solution (so called problem of productivity) and its practical usage [10, 11]. In our case it is interesting to consider the double natured variant of (1) that can be used for the prognostication of the prices.

2. The model of the pricing with linear intersectoral links

Let us consider the input-output model in valuable form "by columns"; moreover the first and the third quadrant. For this we should introduce such indicators:

$\tilde{x}_j^{(1)}$ – the cost of main good from sector *j*;

$\tilde{z}_j^{(1)}$ – the cost of net output from the main sector *j*;

$\tilde{x}_s^{(2)}$ – the cost of the destroyed contaminant *s*;

$\tilde{z}_s^{(2)}$ – the cost of net output from the support sector *s*;

$\tilde{x}_{ij}^{(11)}$, $\tilde{x}_{il}^{(12)}$, $\tilde{x}_{lj}^{(21)}$, $\tilde{x}_{ls}^{(22)}$ – appropriate valuable analogues of the above costs

$A_{11} = \left(a_{ij}^{(11)}\right)_{i,j=1}^{n}$, $A_{12} = \left(a_{il}^{(12)}\right)_{i,l=1}^{n,m}$ and outputs $A_{21} = \left(a_{lj}^{(21)}\right)_{l,j=1}^{m,n}$, $A_{22} = \left(a_{ls}^{(22)}\right)_{l,s=1}^{m}$.

In accordance with the introduced indicators the system of balanced correlation can be written as follows:

$$\tilde{x}_j^{(1)} = \sum_{i=1}^{n} \tilde{x}_{ij}^{(11)} + \sum_{l=1}^{m} \tilde{x}_{lj}^{(21)} + \tilde{z}_j^{(1)}, \quad j = \overline{1,n}, \tag{2}$$

$$\tilde{x}_j^{(2)} = \sum_{i=1}^{n} \tilde{x}_{is}^{(12)} + \sum_{l=1}^{m} \tilde{x}_{ls}^{(22)} + \tilde{z}_s^{(2)}, \quad s = \overline{1,m}\,. \tag{3}$$

The equalities (2) show that the cost of products j $\left(j = \overline{1,n}\right)$ equals the sum of valuable costs that we use for the needs of main activity: production, destroying of pollutants and net output; the equalities (3) – the structure of the cost of destroying contaminants s $\left(s = \overline{1,\, m}\right)$ that is determined by valuable costs of the production in main activity, destroying pollutants that appears during destruction of pollutant s (of secondary activity) and net output from the support sector.

The system of relations (2) and (3) actually formalize double-natured variant of formula (1) by valuable analogue. Assume $p_j^{(1)}$—the price of one good *j*, $p_s^{(2)}$—the price of destroying of contaminant *s* in number 1, $k_j^{(1)}$—coefficient of net output at one good *j*, $\left(\tilde{z}_j^{(1)} = k_j^{(1)} x_j^{(1)}\right)$ or relative price of main product in amount 1 that included in net output from sector *j* of main production (if $\hat{k}_j^{(1)}$—portion of good *j* that included in net output than $k_j^{(1)} = \hat{k}_j^{(1)} p_j^{(1)}$ and $\tilde{z}_j^{(1)} = k_j^{(1)} x_j^{(1)} = p_j^{(1)}\left(\hat{k}_j^{(1)} x_j^{(1)}\right)\Big)$; $k_s^{(2)}$ coefficient of net output from support sector s $\left(\tilde{z}_s^{(2)} = k_s^{(2)} x_s^{(2)}\right)$ or relative price of destroying contaminant *s* that included in net output from sector *s* of support production (if $\hat{k}_s^{(2)}$ – portion of the destroyed contaminant that included in net output than $k_s^{(2)} = \hat{k}_s^{(2)} p_s^{(2)}$ i $\tilde{z}_s^{(2)} = k_s^{(2)} x_s^{(2)} = p_s^{(2)}\left(\hat{k}_s^{(2)} x_s^{(2)}\right)\Big)$.

In this way we can rewrite the valuable components of correlations (2) and (3):

$$\tilde{x}_j^{(1)} = p_j^{(1)} x_j^{(1)}, \quad \tilde{x}_s^{(2)} = p_s^{(2)} x_s^{(2)}, \tag{4}$$

$$\tilde{x}_{ij}^{(11)} = a_{ij}^{(11)} x_j^{(1)} p_i^{(1)}, \quad \tilde{x}_{is}^{(12)} = a_{is}^{(12)} x_s^{(2)} p_i^{(1)}, \quad \tilde{x}_{lj}^{(21)} = a_{lj}^{(21)} x_j^{(1)} p_l^{(2)}, \tag{5}$$

$$\tilde{x}_{ls}^{(22)} = a_{ls}^{(22)} x_s^{(2)} p_l^{(2)}, \quad z_j^{(1)} = k_j^{(1)} x_j^{(1)}, \quad \tilde{z}_s^{(2)} = k_s^{(2)} x_s^{(2)}. \tag{6}$$

Dividing the first *n* equalities of the system (4) appropriately into $x_j^{(1)} > 0$ and the next *m* equalities into $x_s^{(2)} > 0$, we will get the system:

$$\begin{cases} p^{(1)} = A_{11}^T p^{(1)} + A_{21} p^{(2)} + k^{(1)}, \\ p^{(2)} = A_{12}^T p^{(1)} + A_{22}^T p^{(2)} + k^{(2)}. \end{cases} \quad (7)$$

Here is the same problem of productivity for model (7), namely the existence of positive solution of $p^{(1)}$, $p^{(2)}$ with predefined matrixes A_{11}, A_{12}, A_{21}, A_{22} and vectors $k^{(1)}$, $k^{(2)}$. As the model (7) can be written in the view

$$p = A^T p + k , \quad (8)$$

where $p = \left(p^{(1)},\, p^{(2)}\right)^T$, $k = \left(k^{(1)},\, k^{(2)}\right)^T$,

$$A = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix},$$

so the problem of productivity of the model (7) (or (8)) is the same as for Leontief model [21] therefore we will not consider it.

Now consider the problem of pricing, which can be sold on the base of model (2), (3) and (7) using some approaches. Almost all the approaches are based on the assumptions concerning the methods of calculations of the net output, namely concerning the structure of the net output vectors. Denoting by $\alpha^{(1)}, \beta^{(1)}, \gamma^{(1)} \in \mathbb{R}_+^n$ and $\alpha^{(2)}, \beta^{(2)}, \gamma^{(2)} \in \mathbb{R}_+^m$ —appropriate vectors of such coefficients as amortization, salaries and additional product of main and support sectors $\left(k^{(1)} = \alpha^{(1)} + \beta^{(1)} + \gamma^{(1)},\ k^{(2)} = \alpha^{(2)} + \beta^{(2)} + \gamma^{(2)}\right)$ and assuming, that additional product is proportional to salaries with coefficient υ (υ—the only rule of additional product), we will go from the system (6) to system:

$$\begin{cases} p^{(1)} = A_{11}^T p^{(1)} + A_{21} p^{(2)} + \alpha^{(1)} + (1+\nu)\beta^{(1)}, \\ p^{(2)} = A_{12}^T p^{(1)} + A_{22}^T p^{(2)} + \alpha^{(2)} + (1+\nu)\beta^{(2)}. \end{cases} \quad (9)$$

Sometimes from system (9) researchers pass to simplified system

$$\begin{cases} p^{(1)} = \overline{A}_{11}^T p^{(1)} + \overline{A}_{21}^T p^{(2)} + (1+\nu)\beta^{(1)}, \\ p^{(2)} = \overline{A}_{12}^T p^{(1)} + \overline{A}_{22}^T p^{(2)} + (1+\nu)\beta^{(2)}, \end{cases} \quad (10)$$

where amortization is divided by material composition and added to appropriate coefficients of blocked matrix

$$\overline{A} = \begin{pmatrix} \overline{A}_{11} & \overline{A}_{12} \\ \overline{A}_{21} & \overline{A}_{22} \end{pmatrix},$$

where $\overline{A}_{21} = A_{21}$, $\overline{A}_{22} = A_{22}$.

In any case ((7), (9) and (10)) the finding of the solution of balanced prices thros together to the solving of the appropriate system of linear algebraic equations that have a big dimension. In particular the solution of (10) explicitly is:

$$p^{(1)} = (1+\nu)\left[\left(I_n - \overline{A}_1^T\right)^{-1}\beta^{(1)} + \left(I_n - \overline{A}_1^T\right)^{-1}\overline{A}_{21}^T\left(I_m - \overline{A}_{22}^T\right)^{-1}\beta^{(2)}\right],$$

$$p^{(2)} = (1+\nu)\left[\left(I_m - \overline{A}_2^T\right)^{-1}\overline{A}_{12}^T\left(I_n - \overline{A}_{11}^T\right)^{-1}\beta^{(1)} + \left(I_m - \overline{A}_2^T\right)^{-1}\beta^{(2)}\right],$$

where

$$\overline{A}_1^T = \overline{A}_{11}^T + \overline{A}_{21}^T\left(I_m - \overline{A}_{22}^T\right)^{-1}\overline{A}_{12}^T,$$

$$\overline{A}_2^T = \overline{A}_{22}^T + \overline{A}_{12}^T\left(I_n - \overline{A}_{11}^T\right)^{-1}\overline{A}_{21}^T,$$

I_n та I_m – single diagonal matrix with dimension ($n \times n$) and ($m \times m$).

The problem of the pricing is not only difficult and essential for any changes in economy (or ecological-economy) system, as the determining, for example on the base of models (9), (10), balanced prices does not mean that these prices will be really in practice. Besides, the changes at least in one component of the price vector $p^{(1)}$ or $p^{(2)}$ break the balancing in the set of prices. In order to access from one set of balanced prices to another set it is necessary to use the price indexes. It is proven that the problem of determining the price indexes can be solved using the double-natured model.

For determining the price indexes let us back to question of the structure of net output vectors. It is clear that except amortization, salaries and additional product in this structure can be present also other components. Without writing its economic sense purpose that

$$\tilde{z}_j^{(1)} = \sum_{r=1}^{R}\tilde{\upsilon}_{rj}^{(1)} = \sum_{r=1}^{R}\upsilon_{rj}^{(1)}p_j^{(1)}, \quad j = \overline{1,\ n}, \tag{10}$$

$$\tilde{z}_s^{(2)} = \sum_{q=1}^{Q}\tilde{\upsilon}_{qs}^{(2)} = \sum_{q=1}^{Q}\upsilon_{qs}^{(2)}p_s^{(2)}, \quad s = \overline{1,\ m}, \tag{11}$$

where $\tilde{\upsilon}_{rj}^{(1)}$—the cost of net output with type *r* from sector *j* of main industry;

$\upsilon_{rj}^{(1)}$—the amount of net output with type *r* from sector *j* of main industry in kind;

$\tilde{\upsilon}_{qs}^{(2)}$—the cost of net output with type *q* from sector *s* of support industry;

$\upsilon_{qs}^{(2)}$—the cost of net output with type *q* from sector *s* of support industry in kind.

The system of equations (2) and (3) considering (11) and (12) get the system of balanced interrelations

$$x_j^{(1)} p_j^{(1)} = \sum_{i=1}^{n} x_{ij}^{(11)} p_i^{(1)} + \sum_{l=1}^{m} x_{lj}^{(21)} p_l^{(2)} + \sum_{r=1}^{R} \upsilon_{rj}^{(1)} p_j^{(1)}, \;\; j = \overline{1, n}, \tag{13}$$

$$x_s^{(2)} p_s^{(2)} = \sum_{i=1}^{n} x_{is}^{(12)} p_i^{(1)} + \sum_{l=1}^{m} x_{ls}^{(22)} p_l^{(2)} + \sum_{q=1}^{Q} \upsilon_{qs}^{(2)} p_s^{(2)}, \;\; s = \overline{1, m}. \tag{14}$$

Denote by $\pi^{(1)} = \left(\pi_1^{(1)}, \dots, \pi_n^{(1)}\right)^T \in \mathbb{R}_+^n$, $\pi^{(2)} = \left(\pi_1^{(2)}, \dots, \pi_m^{(2)}\right)^T \in \mathbb{R}_+^m$—appropriate vectors of price indexes for the next time of period $[t, \, t+1]$ in the comparison of previous time of period $[t-1, \; t]$. On that, for providing the balance of cost we ought to go from (13) and (14) to

$$x_j^{(1)} p_j^{(1)} \pi_j^{(1)} = \sum_{i=1}^{n} x_{ij}^{(11)} p_i^{(1)} \pi_i^{(1)} + \sum_{l=1}^{m} x_{lj}^{(21)} p_l^{(2)} \pi_l^{(1)} + \sum_{r=1}^{R} \upsilon_{rj}^{(1)} p_j^{(1)} \pi_j^{(1)}, \;\; j = \overline{1, n}, \tag{15}$$

$$x_s^{(2)} p_s^{(2)} \pi_s^{(2)} = \sum_{i=1}^{n} x_{is}^{(12)} p_i^{(1)} \pi_i^{(2)} + \sum_{l=1}^{m} x_{ls}^{(22)} p_l^{(2)} \pi_l^{(2)} + \sum_{q=1}^{Q} \upsilon_{qs}^{(2)} p_s^{(2)} \pi_s^{(2)}, \;\; s = \overline{1, m}. \tag{16}$$

In turn equations (15), (16), obviously can be written as:

$$\tilde{x}_j^{(1)} \pi_j^{(1)} = \sum_{i=1}^{n} \tilde{x}_{ij}^{(11)} \pi_i^{(1)} + \sum_{l=1}^{m} \tilde{x}_{lj}^{(21)} \pi_l^{(2)} + \sum_{r=1}^{R} \tilde{\upsilon}_{rj}^{(1)} \pi_j^{(1)}, \;\; j = \overline{1, n}, \tag{17}$$

$$\tilde{x}_s^{(2)} \pi_s^{(2)} = \sum_{i=1}^{n} \tilde{x}_{is}^{(12)} \pi_i^{(1)} + \sum_{l=1}^{m} \tilde{x}_{ls}^{(22)} \pi_l^{(2)} + \sum_{q=1}^{Q} \tilde{\upsilon}_{qs}^{(2)} \pi_s^{(2)}, \;\; s = \overline{1, m}. \tag{18}$$

So, taking into the consideration that at period $[t, \, t+1]$ the structure of costs is constant in comparison with the previous period $[t-1, \; t]$, the system (15), (16) (or (17), (18)) can be used for the forecasting price indexes in environmental economics. The forecasts of the price indexes allow control of the prices balances and in time to react to changes in any constituent of net output. From the point of view of the decision making person it allows to optimize the process of pricing and its dynamics.

3. Empirical Analysis

The aim of empirical analysis is the prognostication of price indexes in Ukraine for 2012-2015 years on the base of built models. For this reason we design the application using Matlab software. Its structure is shown at the Fig. 1:

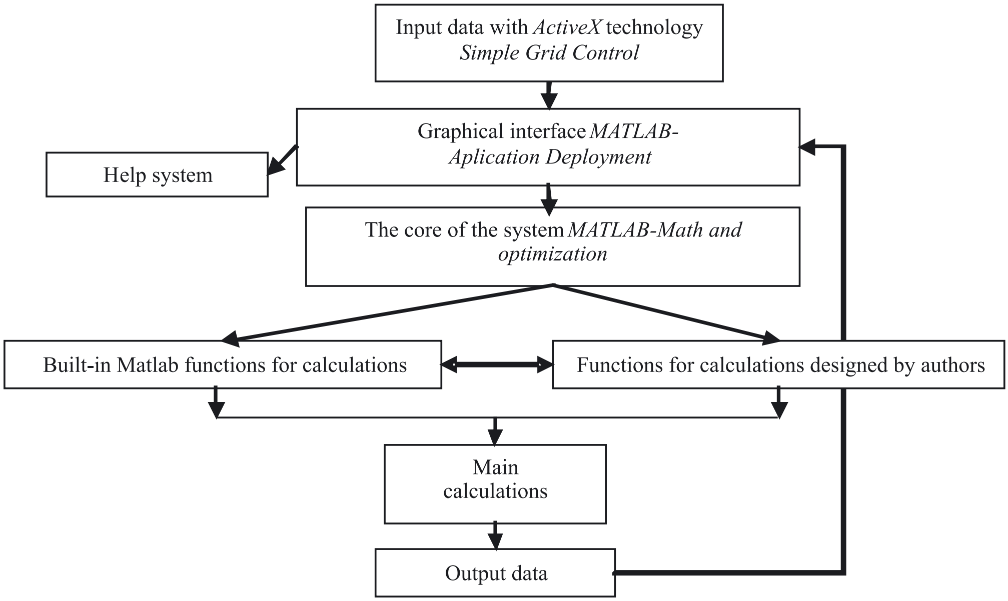

Fig. 1. The structure of the application

The empirical analysis of pricing in environmental economics was made for one supporting sector and such main sectors as (according to classification in Ukraine):

- agriculture, hunting, forestry;
- fishing, fish farming;
- mining industry;
- processing industry;
- production and distribution of electricity, gas and water;
- construction;
- trade, repair of motor vehicles, household goods and personal and household goods;
- hotels and restaurants;
- transport and communication;
- financial affairs;
- real estate, renting and services to individuals;
- public administration;
- education;
- health care and social assistance;
- communal and personal service activities in the field of culture and sport.

On the base of statistical data for the environmental input-output model with linear inter-sectoral links it was get such results:

1. The vectors of main sectors $x^{(1)} \in \mathbb{R}_+^n$ and destroyed industrial contaminant $x^{(2)} \in \mathbb{R}_+^m$ are as follows:

Table 1. The total output of main production and total destroyed industrial pollutants (UAH)

The total output of main production	The total destroyed industrial contaminant
12,8257.32	16,08.2729
1,257.0561	
74,178.722	
802,288.8	
68,339.831	
100,115.3	
174,268.02	
15,779.222	
140,590,59	
59,054.469	
114,523.74	
46,485.135	
46,711.98	
36,367.618	
27,167.807	

If to compare the data that shown in Table 1 with the data that is present in the official Ukrainian input-output tables we can make the conclusions that determined results correctly express the logic of the model (1) and designed software.

2. The prognostic values of net outputs coefficients from main and support industries are calculated according to econometric methods (Table 2).

Table 2. The prognostic values of net outputs coefficients

The net outputs coefficients		Years			
		2012	2013	2014	2015
Main sectors	agriculture, hunting, forestry;	0.359	0.348	0.3339	0.329
	fishing, fish farming;	0.304	0.313	0.321	0.328
	mining industry;	0.545	0.577	0.609	0.641
	processing industry;	0.271	0.283	0.293	0.303
	production and distribution of electricity, gas and water;	0.368	0.354	0.286	0.16
	construction;	0.313	0.294	0.276	0.258
	trade, repair of motor vehicles, household goods and personal and household goods;	0.545	0.539	0.533	0.527
	hotels and restaurants;	0.551	0.54	0.552	0.564
	transport and communication;	0.492	0.473	0.455	0.437
	financial affairs;	0.756	0.769	0.789	0.809
	real estate, renting and services to individuals;	0.514	0.495	0.476	0.457
	public administration;	0.798	0.831	0.862	0.893
	education;	0.702	0.696	0.69	0.684
	health care and social assistance;	0.644	0.651	0.658	0.665
	communal and personal service activities in the field of culture and sport.	–0.366	–0.447	–0.527	–0.607
Support sector	$k_1^{(2)}$	0.913	0.9	0.876	0.829

3. According to obtained coefficients of net outputs and the double-natured Leontief-Ford model (7) we have got such price indexes (table 3):

Table 3. The prognostic values of price indexes

Price indexes		Years			
		2012	2013	2014	2015
Price indexes of goods of main production	agriculture, hunting, forestry;	0.359	0.348	0.3339	0.329
	fishing, fish farming;	0.304	0.313	0.321	0.328
	mining industry;	0.545	0.577	0.609	0.641
	processing industry;	0.271	0.283	0.293	0.303
	production and distribution of electricity, gas and water;	0.368	0.354	0.286	0.16
	construction;	0.313	0.294	0.276	0.258
	trade, repair of motor vehicles, household goods and personal and household goods;	0.545	0.539	0.533	0.527
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	education;	0.702	0.696	0.69	0.684
	health care and social assistance;	0.644	0.651	0.658	0.665
	communal and personal service activities in the field of culture and sport.	–0.366	–0.447	–0.527	–0.607
The price indexes of destroying the industrial contaminant		0.913	0.9	0.876	0.829

Conclusions

The constructed models can be used for prognostication of the price indexes in multi-sectoral eco-economic system. The forecast of the price indexes lets control the prices balances and in time to react on changes in any constituent of net output. From the point of view of the decision making person it allows to optimize the process of pricing and its dynamics in environmental economics.

The information system of uniting of pricing tasks in the ecologically balanced economy is created that contains the kit of visual resources for empirical analysis of developed economic-mathematical models and its usage in experimental researches and monitoring tasks.

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KAINODAROS MODELIAVIMAS APLINKOSAUGOS EKONOMIKOJE

Andrii VERSTIAK, Vasyl GRYGORKIV, Mariia GRYGORKIV

Straipsnyje yra nagrinėjamos ekologinio komponento integravimo kainoje bei būtinybės sudaryti kainodaros modelį, įtraukiant aplinkos ekonomiką, problemos. Straipsnyje pateikti kainodaros modeliai, kuriuos galima pritaikyti sprendimų priėmimui bei kainų indeksų prognozei daugiašakėje aplinkos ekonomikoje. Taikant šį modelį galima realizuoti kainodaros procesus aplinkos ekonomikos sistemoje. Modeliai remiasi matematiniu modeliavimu ir kompiuterinėmis sistemomis, todėl juos galima pritaikyti sprendžiant konkrečias praktines problemas.

Verstiak ANDRII, Phd., Associate professor. Department of Economics, Chernivtsi National University, Ukraine. Research fields: mathematical and quantitative methods in environmental economics.

Verstiak ANDRII, socialinių mokslų daktaras, docentas, Černovicų nacionalinis universitetas, Ekonomikos katedra, Ukraina. Mokslinių tyrimų sritys: matematiniai ir kiekybiniai metodai, aplinkos ekonomika.

Grygorkiv VASYL, Phd., Professor. Department of Economics, Chernivtsi National University, Ukraine. Research fields: mathematical and quantitative methods in environmental economics.

Grygorkiv VASYL, socialinių mokslų daktaras, profesorius, Černovicų nacionalinis universitetas, Ekonomikos katedra, Ukraina. Mokslinių tyrimų sritys: matematiniai ir kiekybiniai metodai, aplinkos ekonomika.

Grygorkiv MARIIA, Research assistant. Department of Economics, Chernivtsi National University, Ukraine. Research fields: mathematical and quantitative methods.

Grygorkiv MARIIA, jaunesnysis mokslo darbuotojas, Černovicų nacionalinis universitetas, Ekonomikos katedra, Ukraina. Mokslinių tyrimų sritys: matematiniai ir kiekybiniai metodai.