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CROSS-COUNTRY COMPARATIVE ANALYSIS FOR THE NATIONAL ECONOMIES BASED ON AGGREGATED INPUT-OUTPUT MODELS

Vasyl GRYGORKIV

Chernivtsi National University
E-mail: v.hryhorkiv@chnu.edu.ua

Andrii VERSTIAK

Chernivtsi National University
E-mail: a.verstyak@chnu.edu.ua

Mariia GRYGORKIV

Chernivtsi National University
E-mail: m.hryhorkiv@chnu.edu.ua

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Abstract. The paper is devoted to the issues of aggregating static input-output models and the usage of appropriate aggregate models in macroeconomic analysis, in particular to compare the aggregated performances of economies of different countries or regions. Investigating economic systems the Input-Output models allow to detect and substantiate the proportions and disproportions in the economy at the different levels of aggregation, and to investigate the sectoral structure of the economies reflected in this paper.

JEL classification: P52, B22, C67, C68.
Keywords: aggregating, economics, pricing, intersectoral balanced models, input-output model, linear models, country comparison, cross-country analysis.
Reikšminiai žodžiai: agregavimas, ekonomika, kainodara, tarpsektorinis modelis, tarpšakinio balanso modelis, linijinis modelis, šalių palyginimas, palyginimas tarp šalių grupių.

Introduction

Input-Output models of different countries have different dimensions and the range of industries, thus for comparative macroeconomic analysis of the production and distribution processes it is advisable to aggregate these models to models of equal dimension, but on the macro level - to models of those formalised by one equation. The aggregation of models to one equation must be implemented where the vector of equilibrium prices is associated as the aggregation operator; the so-called dual-balance model is built relatively to the vector of equilibrium prices. In the aggregation of static input-output model, the ratio of exact aggregation and conditions for technological matrices must be used, which ensure the existence of exact aggregation operator. Note that problem of aggregation comes down to an almost-known scientific problem - to overcome high dimensionality of the original problem by replacing a simple (aggregate) task. On the one hand, no model can fully reproduce all the elements of the system, the links between them and with the environment, that is from the beginning model in a sense is the aggregate, and on the other hand, it is not necessary, since the goal of the research is to integrate economic indicators, combining them into a single group or subgroup. In the transition from lower to higher levels of management, performance indicators are always aggregated, and their number is reduced, although some of the original information is lost and the subsequent calculation is often approximate.

With respect to aggregation in input-output tables it should be noted that it will *a priori* combine different products into generalised product manufactures in the industry and so on. Thus, the association of products can be made in the technological sequence or similarity between them in terms of economic or technical purpose of production conditions. However, the most rational ways of aggregating products and industries are those used today relatively to prices. This is very important for high-quality natural and financial analysis, as indicators, such as rates of financial stability, capital turnover, liquidity, etc. are based on the aggregated balance sheets

Based on the Input-Output statistics of several countries, we will try to build concrete aggregate univariate models, reflecting the distribution of gross domestic product at the intermediate and final products, specifying the value of each component in the balance sheet. The performance analysis of the production systems for each country has been made on the basis of statistical data and the aggregated balance models. The deviation between these performances can determine the limits of possible improvement of aggregate factors in a certain economy, the efficiency of each economy and make a comparative analysis of the economies in the case of equilibrium and the actual prices that in turn can detect and substantiate some of their features.

We would like to remark the papers belonging to the studied problem: Hatanaka, 1952; Ara, 1959; Malinvaud, 1954; Kossov, 1972; Morimoto, 1971, who analysed the problem setting the conditions to be accomplished by industry production functions, so that, when aggregating them into sectors, technical coefficients defined on the basis of sectors are not affected by variations in the final demand vector. We also want to note Theil, 1957, who has established the conditions for zero first order bias, which implies to group industries into almost input-homogeneous sectors. There are other methods that try to minimise the bias (Kymn and Norsworthy, (1976), but most of them, as those that have already been mentioned, take the aggregation scheme for granted (Fisher, 1969; Neudecker, 1970).

1. Methodology

Common aggregation algorithm does not exist. It is usually used as an aggregation operator vector or matrix of equilibrium prices for products. In the case of the classical input-output model (Leontief model)

$$x = Ax + y \tag{1}$$

where $x = (x_1,..., x_n)^T$ and $y = (y_1,..., y_n)^T$ - vectors of gross and final outputs respectively (T - transpose operation). $A = \left(a_{ij}\right)_{i,j=1}^{n}$ - matrix of coefficients of direct material costs (technological matrix), n - number of sectors, aggregation operator is the vector $p = (p_1,..., p_n)^T$ (p_i - price of product i), which allows the aggregate model (1) to the one-dimensional equation

$$x^* = \alpha x^* + y^* \tag{2}$$

where $x^* = p^T x$, $y^* = p^T y$, α - numerical ratio of direct production costs. Value for exact aggregation is given by the so-called Hatanaka condition (Hatanaka, 1952)

$$p^T A = \alpha p^T. \tag{3}$$

In the case of productive indecomposable matrix *A* (Grygorkiv, 2009; Onushenko, 2011) and positive *y* there is always an exact aggregation (2) (ie, the condition (3)), were y^* is a positive number, $\alpha = \lambda_A < 1$ - the actual value and $p^T = p_A^T$ - left Frobenius vector of matrix *A* (Grygorkiv, 2009).

Based on statistical input - output data, we can solve the problem of aggregation model (1) to the one-dimensional equation (2). According to input-output tables, we can find vector final outputs *Y* and technological matrix *A*, then $x = (I - A)^{-1} y$. Confidence factor α is the Frobenius root of matrix *A*. Finding α, we define the aggregated vector $p^T = p_A^T$ from a system of relations:

$$p_A^T (A - \alpha I) = 0. \tag{4}$$

In equation (4) matrix *I* - diagonal identity matrix of dimension $(n \times n)$. Vector p_A^T as the solution of system (4) is the vector of equilibrium prices for the investigation of the production system (product price is proportionate to its production costs). Thus, the vector p_A^T is taken in normalised form $\left(\left\|p_A^T\right\| = 1\right)$. Then, we multiply (1) on the left by the vector p_A^T and obtain the aggregated input-output model (2).

2. Data selection

Input-Output tables describe the sale and purchase relationships between producers and consumers within an economy. They can be produced by illustrating flows between the sales and purchases (final and intermediate) of industry outputs or by illustrating the sales and purchases (final and intermediate) of product outputs. The OECD Input-Output database is presented on the former basis, reflecting in part the collection mechanisms for many other data sources, such as research and development R&D expenditure data, employment statistics, pollution data, energy consumption, which are mainly collected by enterprise or by establishment, and thus according to industry classifications (OECD, 2014).

The latest set of OECD Input-Output tables includes matrices of inter-industrial flows of transactions of goods and services (domestically produced and imported) in current prices, for all OECD countries (except Iceland) and 15 non-member countries, covering the years 1995, 2000 and 2005 or nearest years. Tables for four countries (Cyprus, Malta, Latvia and Lithuania) have been recently added as well as an estimated table for the EU as a whole. Through the use of a standard industry list based on ISIC Revision 3, comparisons can be made across countries. Further information for each country and the estimation methodology is available in the document OECD Input-Output Database edition 2006 - STI Working Paper 2006/8 (OECD, 2014).

The database is a very useful empirical tool for economic research and structural analysis at international level. It highlights inter-industrial relationships and covers not only manufacturing but also services. When used in conjunction with other OECD databases on industrial structures, such as STAN Industry Database (STAN), STAN Bilateral Trade Database by Industry and End-Use (BTDIxE) and the STAN Business R&D Expenditures by Industry (ANBERD), it provides a tool for consistent economic analysis of growth, structural change, productivity, competitiveness and employment at both sectoral and macroeconomic levels (e.g. Productivity Growth in Services Industries) (OECD, 2014). In this paper we analyse the summary input-output table for EU-27, separately for Lithuania, Latvia and Estonia plus Ukraine for the nearest years.

3. Empirical evidence

Specified equation (2) allows assessing the activity of the country or region at the macroeconomic level in some way, because it includes the value of gross output and the final output, and hence determines the amount of the gross domestic production in the intermediate and final production. This allows analysing the performance of the production system and comparing it with the real input - output statistics. Note also that the aggregation operator is the vector of equilibrium, rather than the actual price, so these vectors differ from each other. Vector of equilibrium prices is closely linked to technological matrix A, changes in production technology lead to a change in the equilibrium vector, the use of which in aggregation can eliminate the impact of price fluctuations inherent in real prices, the aggregate size and productivity. The difference between the performance of the production system, obtained by analysing the statistics and aggregated balance model indicates the limits of change in prices for the products, which in many cases can improve the quality of aggregates studied in the economic system.

From equation (4) we obtained that the aggregate rate $\tilde{\alpha}$ of intermediate consumption is the Frobenius number of a proper technological matrix in Leontief model. In Table 1 we have got the specific value of the coefficient $\tilde{\alpha}$ for the EU (separately for Lithuania, Latvia and Estonia) and Ukraine found for their technological matrices.

Table 1. Ratio of intermediate consumption

Country	$\tilde{\alpha}$
EU-27	0.545
Lithuania	0.494
Latvia	0.584
Estonia	0.688
Ukraine	0.566

Source: Author's calculations

Using the value $\tilde{\alpha}$ and solving the system of linear homogeneous equations (4), we obtain the coordinates of the aggregation vector p_A^T that up to a scalar factor is a vector of equilibrium prices for the relevant economic system where the price of production is proportionate to the cost of its production. The final vector p_A^T (aggregation operator) is chosen in normalised form $\left(\left\|p_A^T\right\|=1\right)$.

In Table 2 we have got the coordinates of the normal vector p_A^T for the studied countries.

Table 2. Normalised vector of aggregation

Country	p_A^T
EU-27	0,13 0,06 0,06 0,06 0,11 0,16 0,21 0,10 0,15 0,08 0,31 0,16 0,06 0,01 0,05 0,02 0, 01 0.01 0.04 0.08 0.21 0.18 0.50 0.02 0.42 0.10 0.29 0.10 0.01 0.02 0.00 0.22 0.01 0 , 01 0.00 0.03 0.24
Lithuania	0,12 0,36 0,07 0,12 0,13 0,16 0,06 0,11 0,28 0,04 0,20 0,11 0,12 0,03 0,07 0,09 0, 03 0.05 0.02 0.11 0.38 0.06 0.44 0.04 0.37 0.13 0.13 0.13 0.02 0.02 0.00 0.25 0.01 0 , 02 0.00 0.09
Latvia	0.05 0.00 0.14 0.36 0.06 0.01 0.16 0.01 0.11 0.05 0.09 0.01 0.00 0.00 0.01 0.19 0 19 0,19 0,19 0,19 0,19 0,19 0,19 0,19 0,19 0,19 0,19 0,19 0,19 0,19 0,19 0,19 0,19 0 19 0.19 0.19
Estonia	0,04 0,08 0,02 0,04 0,04 0,06 0,06 0,23 0,09 0,04 0,41 0,24 0,10 0,04 0,46 0,46 0, 03 0.03 0.02 0.02 0.11 0.04 0.19 0.01 0.35 0.10 0.08 0.12 0.04 0.03 0.00 0.22 0.01 0 , 01 0.00 0.02
Ukraine	0.102 0.001 0.303 0.803 0.124 0.014 0.350 0.015 0.254 0.106 0.195 0.018 0.002 0.003 0.024

Source: Author's calculations

Multiplying both sides of the Leontief model (1) by the vector p_A^T, we obtain the aggregated input-output model. Aggregated to one-dimensional equations of balance input-output model for the countries were reduced to ratios specified in Table 3.

Table 3. Aggregate value of the input-output model

Country	Value
EU-27	4314866 = 0,545 · 4541080 + 1841095
Lithuania	7873,433 = 0,494 · 10054,5 + 2904,593
Latvia	1336,843 = 0,584 · 1107,932 + 689,318
Estonia	4233,441 = 0,688 · 4567,203 + 1092,866
Ukraine	1084,624 = 0,566 · 1188,688 + 411,324

Source: Author's calculations

Aggregated balance ratio is a convenient tool for analysing the performance of the national economy at the macroeconomic level, since it contains at once the value of gross domestic production and the value of final output. It is also clear that, according to Table 3, the share of intermediate production is: 54.5% (EU-27), 49.4% (Lithuania), 58.4% (Latvia), 68.8% (Estonia) , 56.6% (Ukraine), and the efficiency of production system is: 45.5% (EU-27), 50.6% (Lithuania), 41.6% (Latvia), 31.2% (Estonia), 43.4% (Ukraine). According to the statistical input-output tables the share of intermediate consumption in real prices and the performance of the national economies and EU-27 was specified (Table 4).

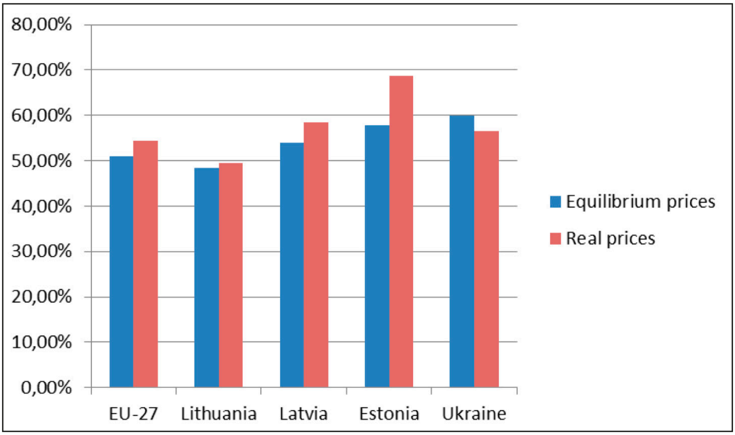
Table 4. The structure of economic systems

Country	The share of intermediate consumption	The performance of the production system
EU-27	50.97%	49.03%
Lithuania	48.37%	51.63%
Latvia	54.01%	45.99%
Estonia	57.91%	42.09%
Ukraine	59.93%	40.07%

Source: Author's calculations

Once again we would like to note that during the procedure of aggregation the vector of equilibrium prices is used, which in general is different from the actual price vector. Since the vector of equilibrium prices is calculated on the basis of the technology matrix A, the obtained price vector changes then and only then when the technological way of production changes. Using vector of equilibrium pricing allows eliminating the impact of price fluctuations, those are largely characteristic of vector of real prices, the aggregate size and the performance indicators of economic activity. In order to compare the results of aggregation of input-output models, let us represent the diagram of the products that are consumed in the production for each country in equilibrium and real terms (Figure 1).

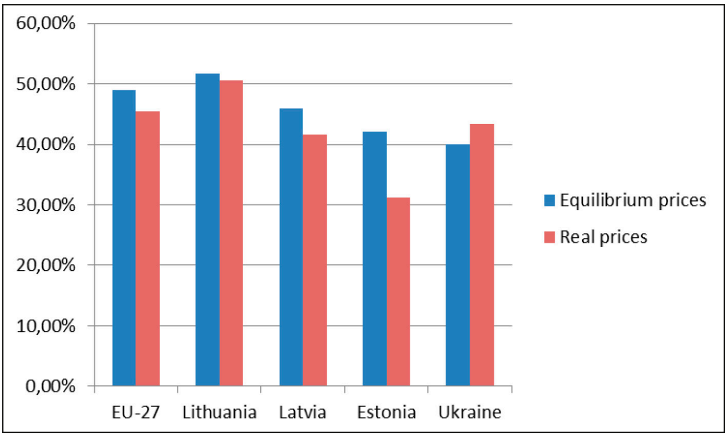
Figure 1: The share of intermediate consumption per capita



Source: Author's calculations

Similarly, we can illustrate the performance of national economies in equilibrium and real terms (Figure 2).

Figure 2. Economic productivity



Source: Author's calculations

The obtained results show that Ukraine's economy is the most material-intensive. For the production of the final product, over 56% of the volume of gross output is consumed in equilibrium prices and almost 60% in real terms. This can be explained by obsolete equipment in businesses activity and the need of its updating and switching to more fuel-efficient technologies. The largest efficiency indicator is in Lithuania.

Conclusion

The proposed aggregation procedures allow making a comparative analysis of different economic systems, working out the optimal pricing policy and balancing the range of fuel and emitted gross output.

The proposed study shows that the difference of intermediate consumption in actual and equilibrium prices is higher for those countries that are less involved in the global market and is less for the countries that are significant players in the global market. In other words, the level of openness and development of market mechanisms in economic regulation determines the appropriate level of deviations between actual and equilibrium prices, which in turn makes it possible to estimate the closeness of an economic system to a state of equilibrium.

It must also be noted that the comparative analysis of the economies of various countries enables aggregation procedure and it is quite simple to determine the performance of the economy and the share of production that is consumed in production.

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ŠALIŲ EKONOMIKOS LYGINAMOJO ANALIZĖ TAIKANT AGREGUOTUS TARPŠAKINIO BALANSO MODELIUS

Santrauka. Straipsnis yra skirtas statinių tarpšakinio balanso modelių agregavimui ir jų panaudojimui makroekonominėje analizėje, o tiksliau siekiant palyginti šalių ar regionų ekonomikos funkcionavimą. Tarpšakinio balanso apibendrintų modelių naudojimas makroekonominėje analizėje visų pirma leidžia palyginti šalių ekonomikos proporcijas ir disproporcijas skirtinguose agregavimo lygiuose ir ištirti sektorinę ekonomikos struktūrą atskirose šalyse ir regionuose. Šiame straipsnyje pateikti tokio atlikto tyrimo rezultatai

Vasyl GRYGORKIV– Prof. Dr., Chernivtsi National University, Ukraine, Department of Economics.

Research interests: mathematical and quantitative methods in environmental economics.

Vasyl GRYGORKIV– Ekonomikos katedros mokslų daktaras, profesorius. Chernivtsi nacionalinis universitetas, Ukraina.

Mokslinių interesų sritys: matematikos ir kiekybinių metodų taikymas aplinkos ekonomikoje.

Andrii VERSTIAK – Chernivtsi National University, Ukraine, Department of Economics, PhD, Associate Professor.

Research interests: mathematical and quantitative methods in environmental economics.

Andrii VERSTIAK – Ekonomikos katedros mokslų daktaras, docentas. Chernivtsi nacionalinis universitetas, Ukraina.

Mokslinių interesų sritys: matematikos ir kiekybinių metodų taikymas aplinkos ekonomikoje.

Mariia GRYGORKIV – Research assistant, Chernivtsi National University, Ukraine, Department of Economics.

Research interests: mathematical and quantitative methods.

Mariia GRYGORKIV – Ekonomikos katedros tyrėja, Chernivtsi nacionalinis universitetas, Ukraina.

Mokslinių interesų sritys: matematika ir kiekybiniai metodai.