ISSN 1822-8011 (print)
ISSN 1822-8038 (online)
INTELEKTINĖ EKONOMIKA
INTELLECTUAL ECONOMICS
2009, No. 2(6), p. 30–41

MULTI-OBJECTIVE OPTIMIZATION WITH DISCRETE ALTERNATIVES ON THE BASIS OF RATIO ANALYSIS

Willem Karel M. BRAUERS

Faculty of Applied Economics and Institute for Development Policy and Management
University of Antwerp
Birontlaan, 97, B2600 Berchem – Antwerpen, Belgium
E-mail: willem.brauers@ua.ac.be

Edmundas Kazimieras ZAVADSKAS

Department of Construction Technology and Management
Vilnius Gediminas Technical University
Saulėtekio ave. 11, LT 10223 Vilnius, Lithuania
E-mail: edmundas.zavadskas@adm.vgtu.lt

Abstract. On the basis of ratio analysis, two methods are developed for multi-objective optimization with discrete alternatives. The first method is MOORA (Multi-objective Optimization on the basis of Ratio Analysis), which serves as a matrix of responses to the alternatives to the objectives to which ratios are applied. The set of ratios has the square roots of the sum of the squared responses as denominators. These ratios are considered to be the best choice among different examples of ratios. The final results varying between zero and one are added up or subtracted to minimize the objective. Finally, all alternatives are ranked according to the size of their obtained dimensionless numbers. Eventually, the most refined way to give more importance to an objective is to replace an objective by different sub-objectives. As a second method, based on the same reasoning as the MOORA method, the Reference Point Method is applied. Ultimately, it is concluded that the second method only serves as a control instrument or as the second best method.

JEL Classification: C02, D81
Keywords: multi-objective optimization, alternative measurement, discrete alternatives, ratio analysis, sub-objectives, MOORA, reference point method, TOPSIS.
Reikšminiai žodžiai: daugiatikslis optimizavimas, alternatyvus matavimas, diskrečiosios alternatyvos, santykių analizė, papildomi tikslai, MOORA, atskaitos taško metodas, TOPSIS.

1. Introduction

1.1. Definitions

The issue of utility has always been crucial for researchers in multi-objective decision making, the starting formula being:

$$max\ U(y) = f[u_1(x_1), u_2(x_2), \ldots, u_n(x_n)] \quad (1)$$

Utility can solely be measured in monetary terms (like in cost-benefit analysis) but researchers consider this attitude to be too materialistic or even unrealistic (Zavadskas, 1990; Roy and Damart, 2005; Brauers, 2004b).

In fact, the utility matter is problematic in four aspects: the choice of units per objective, the normalization, the optimization and the importance given to an objective. In this case the u's can be dropped from the formula.

Nowadays units, attributes, norms, indicators, no matter the name, are used to measure everything, even quality, without any detours to consider the monetary aspects. This is the case in firms, but also in micro- and macroeconomics. Sometimes direct measurement, being too complicated to be made, is substituted by

Alternative Measurement, such as of pollution abatement, quality and individual choice. Let us consider the example of alternative measurement of pollution. Air pollution is difficult to be measured directly. Therefore, pollution abatement costs, for instance, the installation costs in a factory in order to diminish the emission of dangerous gasses and dust represent an alternative measurement of pollution. However, there exist different types of pollution which can be caused by different reasons. For example, air quality in a region like the metropolitan area of Los Angeles could be good, fair or bad. A survey was made in which households in good air quality areas were asked their willingness to pay for a region-wide improvement in air quality. On the basis of the survey and the analysis of the housing market the premium an individual household would have to pay in order to obtain an identical home in a cleaner air region was determined (Brookshire et al., 1982). Regarding noise pollution in residential areas situated near airfields, the cost of complete isolation of houses, the drop in prices of these houses or the amortization of the last models of airplanes would be considered as alternative measurements of noise pollution caused by aircraft noise.

1.2. Assumptions

This article is based on three assumptions.

- *The Assumption of Cardinal Numbers*

As all objectives are assumed measurable in a direct or alternative way, only cardinal numbers are involved (not nominal scales, such as excellent, good, fair or bad).

- *The Assumption of Discrete Choices*

The discrete case counts a number of well-defined and possible alternatives (projects, design). On the contrary, the continuous case generates alternatives during the process itself.

- *The Assumption of Stakeholders*

A decision maker/dictator is replaced by a group of stakeholders. It is not a question of haphazardly choosing one or more decision makers. On the contrary, all stakeholders interested in the issue have to be involved.

1.3. The Methods

Two methods are proposed: firstly, the MOORA method; secondly, the Reference Method with Maximal Criterion Values.

For each method the starting point is a matrix of responses of different alternatives to different objectives:

$$(\boldsymbol{x}_{ij}) \tag{2}$$

with: x_{ij} as the response of alternative j to objective i

$i=1,2,\ldots,n$ as the objectives

$j=1,2,\ldots,m$ as the alternatives

2. The MOORA Method

MOORA is a ratio system in which each response of an alternative to an objective is compared to a denominator which is representative of all the alternatives concerning that particular objective. For this denominator the square root of the sum of squares of each alternative per objective is chosen (Van Delft and Nijkamp, 1977):

$$_N x_{ij} = \frac{x_{ij}}{\sqrt{\sum_{j=1}^{m} x_{ij}^2}} \tag{3}$$

with: x_{ij} = response of alternative j to objective i

$j = 1,2,\ldots,m$; m being the number of alternatives

$i = 1,2,\ldots n$; n being the number of objectives

$_N x_{ij}$ = the normalized response of alternative j to objective i; normalized as a dimensionless number

Dimensionless Numbers, having no specific unit of measurement, are obtained, for instance, by deduction, multiplication or division. The normalized responses of the alternatives to the objectives belong to the interval [0; 1]. However, sometimes the interval could be [-1; 1]. Indeed, for instance, in the case of productivity growth some sectors, regions or countries may show a decrease instead of an increase in productivity, i.e. a negative dimensionless number[1].

For optimization these responses are added in case of maximization and subtracted in case of minimization:

$$_N y_j = \sum_{i=1}^{i=g} {}_N x_{ij} - \sum_{i=g+1}^{i=n} {}_N x_{ij} \tag{4}$$

with: $i = 1,2,\ldots,g$ representing the objectives to be maximized

$i = g+1, g+2,\ldots, n$ representing the objectives to be minimized

$_N y_j$ = the normalized situation of alternative j responding to all objectives

A simulation exercise on privatization illustrates the application of the MOORA and Reference Point methods (Brauers, 2004c, 64).

An ordinal ranking of the $_N y_j$ shows the final preference (Arrow et al., 1949). As Arrow (1974: 256) claims, "a cardinal utility implies an ordinal preference but not vice versa".

[1] Instead of a normal increase in productivity growth a decrease is possible. At that moment the interval becomes [-1, 1]. Consider the example of productivity which has to increase (positive). Consequently, we look into a maximization of productivity, e.g. in European and American countries. What if the opposite variant occurs? For instance, consider the change from USSR to Russia. Contrary to other European countries, productivity decreased. It means that in formula (1) the numerator for Russia would be negative with the whole ratio becoming negative. Consequently, the interval becomes: [-1, +1] instead of [0, 1].

3. Introduction of Ratios in the Reference Point Theory

This method starts from the already normalized ratios as defined in the MOORA method, namely, formula (3).

Concerning the Reference Point Theory a Maximal Criterion Reference Point is chosen, possessing as co-ordinates the highest co-ordinates per objective of all the candidate alternatives. For minimization the lowest co-ordinate is chosen.

In order to measure the distance between the co-ordinates of the alternatives and the reference point, the *Min-Max Metric of Tchebycheff* is chosen (Karlin and Studden, 1966: 280):

$$\underset{(j)}{Min}\left\{\underset{(i)}{\max}\,/\,r_i - {}_N x_{ij}\,/\right\} \qquad (5)$$

with: $i = 1, 2, ..., n$ representing the objectives

$j = 1, 2, ..., m$ representing the alternatives

r_i = the i[th] co-ordinate of the maximal criterion reference point. Each co-ordinate of the reference point is selected as the highest corresponding co-ordinate of the alternatives

${}_N x_{ij}$ = the normalized objective i of alternative j

In the case of a minimum the distances between the rather low co-ordinate of the reference point and the corresponding co-ordinates of the responses of the alternatives to an objective are negative. Therefore, only absolute values are introduced in the Min-Max metric.

The preference for this nonconvex result is necessary in order to respect *Consumer Sovereignty* (further explained in Brauers, 2004b: 132-163).

A simulation exercise illustrates the application of the MOORA method (Table 1).

Table 1. A Simulation of MOORA and the Reference Point Method based on Ratios

1a - Matrix of responses of alternatives to objectives: (x_{ij})

Projects	1. IRR (%) MAX	2. Payback Period (in years) MIN	3. New Inv. (10^9 €) MAX	4. New Employm. (in jobs) MAX	5. V.A. (10^6 €) (discounted) MAX	6. Bal. of Paym. curr. acc. (10^6 €) MAX
Project A	12	5	4.5	750	800	150
Project B	12	7	3	800	600	200
Project C	10	9	2.5	900	850	150
Totals	34	21	10	2,450	2250	500

1b - Sum of squares and their square roots

Projects						
Project A	144	25	20.25	562500	640000	22500
Project B	144	49	9	640000	360000	40000
Project C	100	81	6.25	810000	722500	22500
Sum of squares	388	155	35.5	2012500	1722500	85000
Square roots	19.6977156	12.4498996	5.9581876	1418.6261	1312.4405	291.5475947

1c - Objectives divided by their square roots and MOORA

							sum	rank
Project A	0.609207699	0.401610	0.7552632	0.52868053	0.60955	0.514495755	2.61559	1
Project B	0.609207699	0.562254	0.5035088	0.563926	0.4571636	0.685994341	2.2575	2
Project C	0.507673083	0.722897	0.4195907	0.63441664	0.6476484	0.514495755	2.1560	3

1d - Reference Point Theory with Ratios: co-ordinates of the Reference Point equal to the maximal criterion values

r_i	0.609207699	0.401610	0.7552632	0.63441664	0.64765	0.685994341

1e - Reference Point Theory: deviations from the Reference Point

							max	Rank min
Project A	0	0	0	0.10573611	0.03810	0.171498585	0.17150	1
Project B	0	0.160644	0.2517544	0.070491	0.19048	0	0.251754	2
Project C	0.101534617	0.321288	0.3356725	0	0.00000	0.171498585	0.33567	3

By introducing small changes into the simulation it is shown that the Min-Max Metric of Reference Point Theory with ratios is not flexible enough to react to such changes (Brauers, 2004a: 180). Therefore, this Reference Point Theory is considered to be the second best method (after the MOORA method) or as a control system for MOORA.

In MOORA the choice of the square roots of the sum of the squared responses as denominators may look rather arbitrary (Brauers, 2007a, 2007b; Brauers et. al., 2007; Brauers and Zavadskas, 2006). Therefore, the search for alternative denominators is the main issue of this research.

4. Is the Use of other Denominators in the MOORA Method Advisable?

In the MOORA formula (3) the denominator $\sqrt{\sum_{j=1}^{m} x_{ij}^2}$ was chosen.

Possibilities with other denominators will also be discussed as the following description cannot be said to be exhaustive.

4.1. Voogd (1983) Ratios

$$_N x_{ij} = \frac{x_{ij}}{\sum_{j=1}^{m} x_{ij}} \tag{6}$$

Allen (1951) already used this formula, but Voogd (1983) applied it for multi-objective evaluation. For optimization these responses are added in case of maximization and subtracted in case of minimization (formula (8)).

The total ratios are smaller than those in the square roots method but their calculation is less complicated. However, they will not necessarily lead to the same outcome, e.g. the simulation of marketing in a department store showed different results (Brauers, 2004b: 307-309). Moreover, if many situations similar to the example of productivity occur, the denominator of the ratio could become positive, negative or even equal to zero. Then the ratio itself could obtain all positive or negative values, or could even be undefined. Consequently, the intervals [0; 1] or [-1; 1] are not maintained in the formula of total ratios.

4.2. Schärlig (1985) Ratios

What regards Schärlig Ratios, one of the alternatives is taken as a basis. This mechanical approach is comparable with the formula of Schärlig which multiplies all the fractions.

A problem arises if the alternative which is used as a basis lacks one of the objectives. As a result, some undefined ratios are obtained because the denominator is zero. Therefore, an alternative with no objectives equal to zero has to be chosen as a basis.

Obviously, if another alternative is chosen as a basis, different results are obtained; therefore, a ratio analysis in which one of the alternatives is taken as a basis does not produce a univocal outcome (see simulations in Brauers, 2004b: 297).

4.3. Weitendorf (1976) Ratios

Weitendorf compares the responses with the Maximum-Minimum interval in the following way:

- if $_N x_{ij}$ should be maximized:

$$_N x_{ij} = \frac{x_{ij} - x_i^-}{x_i^+ - x_i^-} \tag{7}$$

- if $_N x_{ij}$ should be minimized:

$$_N x_{ij} = \frac{x_i^+ - x_{ij}}{x_i^+ - x_i^-} \tag{8}$$

with: x_i^+ representing the maximum value and x_i^- representing the minimum value of objective *i*.

The normalized responses belong to the interval *[0; 1]*.

This method which at the first glance seems interesting has to be rejected on the following grounds:

1) the Reference Method with the co-ordinates of the reference point equal to the maximal criterion values cannot be applied as all co-ordinates of the reference point are equal to one (see Table 2e and Table 2f).

2) If only the maximum and the minimum per objective of all alternatives are taken into consideration, the composition of the whole series of objectives remains disregarded, i.e. the following points are not considered:

- the spread as measured by the standard deviation. For several series this spread can be different though with the same maxima and minima;

- the median and the quartiles can be different for several series though with the same maxima and minima.

Therefore, a simulation is made with Weitendorf ratios of the same matrix of responses of alternatives to objectives as in Table 1. In comparison to the square roots ratios presented in Table 1, Table 2 shows the results of the application of the Weitendorf Ratios.

Table 2. Multiple Objectives Optimization with Weitendorf Ratios

2a – Matrix of Responses of Alternatives to Objectives: ($\mathbf{x}_{ij}$)

Projects	1. IRR (%) MAX	2. Payback Period (in years) MIN	3. New Inv. (10^9 €) MAX	4. New Employm. (in jobs) MAX	5. V.A. (10^6 €) (discounted) MAX	6. Bal. of Paym. curr. acc. (10^6 €) MAX
Project A	12	5	4.5	750	800	150
Project B	12	7	3	800	600	200
Project C	10	9	2.5	900	850	150

2b - Responses minus minimum for maximization or maximum minus responses for minimization

Projects						
Project A	2	4	2	0	200	0
Project B	2	2	0.5	50	0	50
Project C	0	0	0	150	250	0

2c – For the denominator: maximum minus minimum

r_i	2	4	2	150	250	50

2d - Data 2b divided by 2c and additive method with Weitendorf ratios

							sum	rank
Project A	1	1	1	0	0.80000	0	1.80000	2
Project B	1	0.50	0.25	0.333333	0	1	2.0833	1
Project C	0	0	0	1	1	0	1.0833	3

2e - Reference Point Theory: co-ordinates of the reference point equal to the maximal criterion values

r_i	1	1	1	1	1	1

2f - Reference Point Theory: deviations from the reference point

							max	rank min
Project A	0	0	0	1	0.20	1	1	1
Project B	0	0.50	0.75	0.666667	1	0	1	1
Project C	1	1	1	0	0	1	1	1

Thousands and thousands of other matrices of responses of alternatives to objectives with the same outcomes of formulae (7) and (8), as given in Tables 2b and 2c, will lead to the same ranking. What is more, the same results would be obtained. For example, the ranking and final results in Table 3 are the same as in Table 2 and have the same relations to their maxima and minima even though the matrix of responses taken as a starting point is different.

Table 3. Multiple Objectives Optimization with Weitendorf Ratios (second trial)

3a – Matrix of Responses of Alternatives to Objectives: (x_{ij})

Projects	1. IRR (%) MAX	2. Payback Period (in years) MIN	3. New Inv. (10^9 €) MAX	4. New Employm. (in jobs) MAX	5. V.A. (10^6 €) (discounted) MAX	6. Bal. of Paym. curr. acc. (10^6 €) MAX
Project A	14	6	7	1000	1200	0
Project B	14	8	5.5	1050	1000	50
Project C	12	10	5	1150	1250	0

3b - Responses minus minimum for maximization or maximum minus responses for minimization

Projects						
Project A	2	4	2	0	200	0
Project B	2	2	0.5	50	0	50
Project C	0	0	0	150	250	0

3c - Maximum minus minimum

r_i	2	4	2	150	250	50

3d - Data 3b divided by 3c and additive method with Weitendorfratios

							sum	rank
Project A	1	1	1	0	0.80000	0	1.80000	2
Project B	1	0.50	0.25	0.333333	0	1	2.0833	1
Project C	0	0	0	1	1	0	1.0833	3

3e - Reference Point Theory: co-ordinates of the reference point equal to the maximal criterion values

r_i	1	1	1	1	1	1

3f - Reference Point Theory: deviations from the reference point

							max	rank min
Project A	0	0	0	1	0.20	1	1	1
Project B	0	0.50	0.75	0.666667	1	0	1	1
Project C	1	1	1	0	0	1	1	1

4.4. Van Delft and Nijkamp (1977) Ratios of Maximum Value

In the method of maximum value the objectives per alternative are divided by the maximum or the minimum value of that objective which are found in one of the alternatives.

$$_N x_{ij} = \frac{x_{ij}}{x_i^+} \qquad (9)$$

with: x_i^+ representing the maximum or minimum x_{ij} depending on whether a maximum or a minimum of an objective is strived for.

As only maxima, minima and the responses are involved here, the same comments on the spread, the median and quartiles as mentioned for the Weitendorf ratios are relevant.

A fundamental problem arises regarding minimization. The ideal situation for minimization occurs when zero is attained. This could mean dividing by zero. If the numerator is not zero the fraction is undefined. Even if in that case a symbolic number, for instance, 0.001 is given to an alternative, for other alternatives the result would be negatively biased. It could solely determine the final ranking of the alternatives, and that is incorrect (an example is given in Brauers, 2004b: 298).

In any case, the ratios can deviate largely from the interval [0; 1]. In this way one of the advantages of the ratio system disappears, namely, the relation between the ratios can differ by one at most.

Once again, when the Reference Point Theory is applied, all co-ordinates of the maximal criterion reference point are equal to one. Indeed, the maximal criterion values are either the maximum value divided by itself or the minimum value divided by itself.

4.5. Jüttler (1966) Ratios

For normalization it is also possible to use Jüttler's ratios:

$$_N x_{ij} = \frac{x_j^+ - x_{ij}}{x_i^+} \qquad (10)$$

As only maxima, minima and the responses are involved here, the remarks on the spread, the median and the quartiles, mentioned earlier, are relevant.

If x_i^+ represents a minimum, as the denominator it can have a zero value. Therefore, the same criticism as against the van Delft and Nijkamp Method of Maximum Value can be expressed.

4.6. Stopp (1975) Ratios

If max x_{ij} is desirable:

$$_N x_{ij} = \frac{100 x_{ij}}{x_i^+} \qquad (11)$$

If min x_{ij} is desirable:

$$_N x_{ij} = \frac{100 x_i^-}{x_{ij}} \qquad (12)$$

These normalized values are expressed in percentages.

As maxima and minima are used, the same criticism as against the Weitendorf Ratios can be expressed.

Hwang and Yoon (1981: 100) mention the same formulae but without percentages.

4.7. Körth (1969 a, b) Ratios

$$_N x_{ij} = 1 - \left| \frac{x_i^+ - x_{ij}}{x_i^+} \right| \qquad (13)$$

Here the same criticism as against the van Delft and Nijkamp Method of Maximum Value and the Weitendorf Ratios can be expressed as well because the maximum value is used.

4.8. Peldschus et al. (1983) and Peldschus (1986, 2007) Ratios for Nonlinear Normalization

If Minimum x_{ij} is desirable:

$$_N x_{ij} = (\frac{x_i^-}{x_{ij}})^3 \quad (14)$$

If Maximum x_{ij} is desirable:

$$_N x_{ij} = (\frac{x_{ij}}{x_i^+})^2 \quad (15)$$

Once again, as only maxima and minima are used, the same arguments as against the Weitendorf Ratios are relevant.

5. The Importance Given to an Objective by the Attribution Method in MOORA

One objective i of $_N x_{ij}$ cannot be significantly more important than the other as all their ratios are smaller than one (see formula (7)). Nevertheless, it may turn out to be necessary to stress that some objectives are more important than others. In order to give more importance to an objective it could be multiplied with a *Significance Coefficient* (for an example see Brauers et al, 2007).

The *Attribution of Sub-Objectives* represents another solution. Consider the example of the purchase of fighter planes (Brauers, 2002). Economically, the objectives concerning the fighter planes are threefold: price, use and balance of payments; however, there is also military effectiveness. In order to give more importance to military defence, effectiveness is reduced by, for instance, giving more importance to the maximum speed, the power of the engines and the maximum range of the plane. Anyway, the Attribution Method is more refined than the Significance Coefficient Method: a more comprehensive characterisation is possible when the Attribution Method is applied. For instance, for employment a significance coefficient of 2 is replaced by two sub-objectives characterizing the direct and the indirect use. In Table 1c for Project A, 2 x 0.52868 is changed by two separate numbers characterizing the direct and the indirect side of employment. However, it is not always easy to find enough sub-objectives on which the stakeholders would agree.

In the Reference Point Theory the Min-Max metric is considered to be the second best method. Is this a correct choice?

6. Is the Min-Max Metric the Best Choice for Reference Point Theory?

6.1. Reference Point Theory as a Very Respectable Theory

The history of the development of the Reference Point Theory is rather long. The foundations of the theory were laid by Tchebycheff (1821-1894) and Minkowski (1864-1909) (see: Karlin and Studden, 1966; Minkowski, 1896, 1911). For further development of the theory significant contributions were made by Benayoun et al. (1971); Wierzbicki (1977, 1980, 1982); van Delft and Nijkamp (1977); Steuer (1989a, 1989b and Steuer and Choo, 1983), Nakayama and Sawaragi (1983), etc. Goal programming represents another development in this sphere; here names such as Lee (1972), Dyer (1972), Tamiz and Jones (1996), etc. must be mentioned.

The choice of the reference point, the distance and the characteristics of the objectives determine the use of the Reference Point Theory (Brauers, 2004b: 156-165).

A method called "TOPSIS" (Technique for Order Preference by Similarity to Ideal Solution) is of particular interest for practitioners (Zavadskas, 1986; Šaparauskas and Turskis, 2006; Zavadskas and Antuchevičienė, 2006).

6.2 I. TOPSIS a Better Choice for the Reference Point Theory?

TOPSIS is a Reference Point Theory which was developed a bit later than the other theories (Hwang and Yoon, 1981: 128).

TOPSIS is "based upon the concept that the chosen alternative should have the shortest distance from the ideal solution" (Hwang and Yoon, 1981: 128) which is, in fact, the aim of every Reference Point Theory or an ideal point, as it is called. The distinction between the TOPSIS method and the MOORA method lies in the definition of distance and in the fact that the ideal point and, ipso facto, each alternative have many co-ordinates corresponding to the number of attributes (a vector). Moreover, an attribute can ask for a maximum or for a minimum attainment. The choice of the distance function and ways to handle maxima and minima make TOPSIS debatable (Zavadskas et. al, 2006; Opricovic and Tzeng, 2004, 2007)

6.2.1. What is Meant by the Shortest Distance?

In TOPSIS the Euclidean distance is chosen to define the shortest distance. Euclidean distances are represented by radii of concentric circles, concentric spheres and, in general, hypersurfaces around the ideal point as a central point. Therefore, according to the definition offered by Minkowski, Euclidean distances are convex: a hypersurface is called convex if it contains with any two points the entire segment joining these two points (Minkowski, 1896: 200; Minkowski, 1911: 103; Pogorelov, 1978: 9). Consequently, in the calculation of Euclidean distances non-convexity (as required by *Consumer Sovereignty)* is disregarded. On the contrary, in the Tchebicheff Min-Max Metric only one (the largest) distance per alternative is kept in the process of calculation; therefore, non-convexity is taken into account.

Calculation of Euclidean distances leads to many similar results. For instance, for the ideal point (100,100) the midway solution (50;50), the extreme positions (100;0) and (0;100) but also (60;40), (40;60), (30;70) and (70;30) have the same Euclidean distances. Even worse, an infinite number of points belonging to the same hypersurface have the same Euclidean distance.

Nevertheless, it is possible that the hypersurfaces are not complete. Everything depends on the philosophy regarding the ideal point. If the ideal point is a *Utopian Criterion Point* no co-ordinate of an alternative can surpass the corresponding co-ordinate of the ideal point. This could be the case in *Performance Management* when the requirements are very high. For instance, in general education the requirements of all subjects of the curriculum could be very high. In the case of choosing marriage candidates the requirements for beauty and cooking could, for instance, be lower but a very high level of intelligence could be required.

If the ideal point is called a *Reference Point* it will have as co-ordinates the highest corresponding co-ordinates of the alternatives. In fact, a reference point is not an optimum point. Therefore, such a situation is sometimes called a *Satisficing Result* or *Bounded Rationality* as it seems that the stakeholders are completely satisfied if the realistic reference point is reached (Wierzbicki, 1982; Ahituv and Spector, 1990). If a new alternative is introduced, the co-ordinates of the reference point could be surpassed. A new reference point could be chosen.

Once again, when the Tchebicheff Min-Max Metric is applied different ideal points can be chosen without difficulties. The only problem may occur if one or more co-ordinates of a newly introduced alternative are larger than those of the existing reference point. In this case the order of the preferences for all alternatives may change. In order to avoid this, the previously established reference point can be maintained, but then negative distances may arise. If, for example, the reference point is r* (15000; 6500; 400) and a new alternative is M (15000; 12000; 0), the deviation for the second co-ordinate will be: - 5500. Therefore, absolute values were introduced in the min-max metric. If this deviation is not allowed, the alternative is fined for 5500 by changing the maximum in a minimum for that response of the alternative to that objective.

6.2.2. How to Handle Maxima and Minima

After normalization and attribution of weights TOPSIS proposes two kinds of reference points: a positive and a negative. The positive reference point has as co-ordinates the highest corresponding co-ordinates of the alternatives (the lowest in case of a minimum). The negative reference point has as co-ordinates the lowest corresponding co-ordinates of the alternatives (the highest in the case of a minimum). With regard to these two kinds of reference points Euclidean distances are calculated. Consequently, each alternative will have two outcomes. Let us call them ${}_{N}y_{j+}$ and ${}_{N}y_{j-}$. In order to come to a single solution, TOPSIS proposes the following formula which is rather arbitrarily chosen (Hwang and Yoon, 1981: 128-134):

$$ {}_{N}y_{j} = {}_{N}y_{j-} / {}_{N}y_{j+} + {}_{N}y_{j-} \qquad (16) $$

with: $j = 1,2,...,m$; m representing the number of alternatives

In addition, Opricovic and Tzeng (2004: 450) conclude that the relative importance of the two outcomes is not considered, although it could be a major concern in decision making.

7. General Conclusions

Several alternative solutions to a problem of utility must be suggested. The notion of utility has always been a crucial point for researchers in multi-objective decision making. For us the notion of utility is problematic in four aspects: the choice of units per objective, the normalization, the optimization and the importance given to an objective.

Ratio development can be a full-fledged method for multiple objective optimization. It can also serve as an additive method with ratios for MOORA. Square roots ratios are the most suitable for the Reference Point Method with a Maximal Criterion Reference Point, while the Voogd ratios are the second best. On the basis of mathematical logic and with reference to several simulations all other methods are considered to be of no value.

At first glance the Weitendorf Ratios look promising. Several tests, however, proved that the method is ambiguous. This is understandable as, regarding the minimum and the maximum, the spread, the median and the quartiles can vary and make the series not univocal. In addition, in the Van Delft and Nijkamp Ratios of Maximum Value, in the denominator a zero value as a minimum can appear making the results undefined. The abovementioned remarks are also relevant to the Juttler, Stopp, Körth and Peldschus Ratios. The ratios *à la* Schärlig are of another kind. As one of the alternatives is taken as a reference with each other alternative the outcome will be different. Of course, minima and maxima are very important notions in many fields of sciences. Unfortunately, they are not sufficient to properly characterize and optimize a matrix of responses of alternatives to objectives.

It is taken for granted that every objective can be measured either directly or by alternative measurement. Is a final ranking universally accepted? Arrow is right in claiming that "obviously, a cardinal utility implies an ordinal preference but not vice versa" (1974: 256).

What regards the square roots ratios and the Voogd sum ratios, one objective cannot be very much larger than another as their ratios are smaller than one. However, it may be necessary to consider some objectives more important than others. How is it possible to take this importance into account? The traditional (but not the best) way is to use weights. To many stakeholders it may be difficult to reach an agreement regarding the choice of weights. A solution could be reached by the analysis of objectives in Sub-Objectives.

In the Reference Point Theory preference is given to the Tchebycheff Min-Max Metric. A reference point per objective possesses as co-ordinates the dominating co-ordinates of the candidate alternatives. For minimization the lowest co-ordinates are chosen and that is more logical than in the TOPSIS method. When the TOPSIS method is applied to two kinds of reference points, a maximum and a minimum, are arrived at, thus making the co-ordination of the sets of points extremely difficult.

The following conclusions regarding the MOORA method can be drawn:

1) Square Roots Ratios are chosen as the best choice for MOORA; Voogd Ratios are the second best.

2) Eventually, more importance to an objective is given by weights or by replacing the objective by different sub-objectives.

3) The ratios per alternative for the objectives to be maximized are added. The ratios per alternative for the objectives to be minimized are subtracted. The general total per alternative will compete in a ranking.

4) The ranking is established.

5) The Reference Point Theory with the Min-Max Metric (but not with TOPSIS) will be used as a control instrument.

References

1. Ahituv, N.; Spector, Y. (1990). Evaluating Information under two Criteria: Expected Payoff and Risk, *Working Paper*, No. 70/90, I.I.B.R., Tel Aviv University, Israel, p. 5.
2. Allen, R. G. D. (1951). *Statistics for Economists.* London: Hutchinson's University Library.
3. Arrow, K. J.; Blackwell, D.; Girshick, M. A. (1949). Bages and minimax solutions of sequential decision problems. In: *Econometrica*, 17: 213-244.
4. Arrow, K. J. (1974). General economic equilibrium: purpose, analytic techniques, collective choice. In: *American Economic Review*, 1: 256.
5. Benayoun, R.; De Montgolfier, J.; Tergny, J.; Larichev, O. (1971). Linear programming with multiple objective functions: step method (STEM). In: *Mathematical Programming*, 3: 366-375.
6. Brauers, W. K. (2002). The multiple representation for multiple objectives optimization with an application for arms procurement. In: *Naval Research Logistics*, 49: 327-340.
7. Brauers, W. K. (2004a) Multi-Objective optimization for facilities management, In: *Journal of Business Economics and Management*, 5(4):173-182.
8. Brauers, W. K. (2004b). *Optimization Methods for a Stakeholder Society. A Revolution in Economic Thinking by Multiobjective Optimization*. Boston: Kluwer Academic Publishers.
9. Brauers, W. K. (2004c). Multiobjective optimization (MOO) in privatization. In: *Journal of Business Economics and Management*, 5(2): 59-65.
10. Brauers, W. K. M.; Zavadskas, E. K. (2006). The MOORA method and its application to privatization in a transition economy. In: *Control and Cybernetics*, 35(2): 445-469.
11. Brauers, W. K. M. (2007a). What is meant by normalisation in decision making? In: *International Journal of Management and Decision Making*, 8 (5/6): 445-460.
12. Brauers, W. K. M. (2007b). Normalisation in multiobjective optimization. In: *International Journal of Management and Decision Making*, 8 (5/6): 461-474.
13. Brauers, W. K. M.; Ginevičius, R.; Zavadskas, E. K.; Antuchevičienė, J. (2007). The European Union in a transition economy. In: *Transformations in Business and Economics*, 6(2): 21-49.
14. Brookshire, D. S.; Thayer M. A.; Schulze, W. D.; D'Arge, R. C. (1982). Valuing public good: a

comparison of survey and hedonic approaches. In: *American Economic Review*, 1: 165-177.
15. Dyer, J. S. (1972). Interactive goal programming. In: *Management Science*, 1: 62-70.
16. Hwang, C. L.; Yoon, K. (1981). Multiple attribute decision making, methods and applications. In: *Lecture Notes in Economics and Mathematical Systems*, p. 186.
17. Jüttler, H. (1966). Untersuchungen zur Fragen der Operationsforschung und ihrer Anwendungsmöglichkeiten auf ökonomische Problemstellungen unter besonderer Berücksichtigung der Spieltheorie. Dissertation A an der Wirtschaftswissenschaftlichen Fakultät der Humboldt-Universität, Berlin.
18. Karlin, S.; Studden, W. J. (1966). *Tchebycheff Systems: with Applications in Analysis and Statistics*. New York: Interscience Publishers.
19. Körth, H. (1969a). *Untersuchungen zur nichtlinearen Optimierung ökonomischer Erscheinungen und Prozesse unter besonderer Berücksichtigung der Quoitientenoptimierung sowie der Lösung ökonomischer mathematischer Modelle bei Existenz mehrerer Zielfuntionen*. Habilitationsschrift Humboldt - Universität Sektion Wirtschaftswissenschaften Berlin.
20. Körth, H. (1969b). Zur Berücksichtigung mehrerer Zielfunktionen bei der Optimierung von Produktionsplanen. In: *Mathematik und Wirtschaft*, 6: 184-201.
21. Lee, S. M. (1972). *Goal Programming for Decision Analysis*. Philadelphia: Auerbach.
22. Minkowsky, H. (1896). *Geometrie der Zahlen*. Leipzig: Teubner.
23. Minkowsky, H. (1911). *Gesammelte Abhandlungen*. Leipzig: Teubner.
24. Nakayama, H.; Sawaragi, Y. (1984). Satisficing trade-off method for multiobjective programming. In: *Lecture Notes in Economics and Mathematical Systems*, 229: 114-122.
25. Opricovic, S.; Tzeng, G. H. (2004). Compromise solution by MCDM methods: a comparative analysis of VIKOR and TOPSIS. In: *European Journal of Operational Research*, 156: 445-455.
26. Opricovic, S.; Tzeng, G. H. (2007). Extended VIKOR method in comparison with outranking methods. In: *European Journal of Operational Research*, 178(2): 514-529.
27. Peldschus, F.; Vaigauskas, E.; Zavadskas, E. K. (1983). Technologische Entscheidungen bei der Berücksichtigung mehrerer Ziele. In: *Bauplanung – Bautechnik*, 37(4): 173-175.
28. Peldschus, F. (1986). *Zur Anwendung der Theorie der Spiele für Aufgaben der Bautechnologie*. Dissertation B, TH Leipzig.
29. Peldschus, F. (2007). The effectiveness of assessment in multiple criteria decisions. In: *International Journal of Management and Decision Making*, 8(5/6): 519-526.
30. Pogorelov, A. V. (1978). *The Minkowski Multidimensional Problem*, Washington D.C: Winston and Sons.
31. Roy, B.; Damart, S. (2005). *Débat publique et Expertise: entre Rationalité et Legitimité.* Cahier No. 222, LAMSADE, Université Paris-Dauphine, Paris.
32. Schärlig, A. (1985). *Décider sur plusieurs critères*. Lausanne : Presses Polytechniques Romandes, p. 76.
33. Steuer, R. E.; Choo, E. (1983). An interactive weighed Tcebycheff procedure for multiple objective programming. In: *Mathematical Programming*, 26: 326-344.
34. Steuer, R. E. (1989a). *Interactive Multiple Objective Programming: Concepts, Current Status and Future Directions.* International Conference on M.C.D.M.: Applications in Industry and Service, Asian Institute of Technology, Bangkok.
35. Steuer, R. E. (1989b). *Multiple Criteria Optimization: Theory, Computation and Application*. Malabar: Krieger Publishing Company.
36. Stopp, F. (1975). Variantenvergleich durch Matrixspiele. In: *Wissenschaftliche Zeitschrift der Hochschule für Bauwesen*, 2: 117.
37. Šaparauskas, J.; Turskis, Z. (2006). Evaluation of construction sustainability by multiple criteria methods. In: *Technological and Economic Development of Economy*, 12(4): 321-326.
38. Tamiz, M.; Jones, D. F. (1996). Goal programming and Pareto efficiency. In: *Journal of Information and Optimization Sciences*, 2: 1-17.
39. Van Delft, A.; Nijkamp, P. (1977). *Multi-criteria Analysis and Regional Decision-making*. Leiden: Martinus Nijhoff Publishers.
40. Voogd, H. (1983). *Multicriteria Evaluation for Urban and Regional Planning*. London: Pion Ltd., p. 86-87.
41. Weitendorf, D. (1976). *Beitrag zur Optimierung der räumlichen Struktur eines Gebäude.* Dissertation A, Hochschule für Architektur und Bauwesen, Weimar.
42. Wierzbicki, A. P. (1977). Basic properties of scalarizing functionals for multiobjective optimization. In: *Mathematische Operationsforschung und Statistik - Series Optimization*,1: 55-60.
43. Wierzbicki, A. P., The use of reference objectives in multiobjective optimization. In: *Lecture Notes in Economics and Mathematical Systems*, 177: 468-486.
44. Wierzbicki, A. P. (1982). A mathematical basis for satisficing decision making. In: *Mathematical Modelling*, 5: 391-405.
45. Zavadskas, E. K. (1986). *The Method of Ranking of Construction-technological Alternatives on the Basis of the Distance from the Ideal Solution. New Construction Technology for Buildings and Structures.* Inter – university selected scientific

papers. Engineering Institute of Consturction: Leningrad, p. 52-57.
46. Zavadskas, E. K. (1990). Variantenauswahl mit der Nutzensfunktion. In: *Wissenschaftliche Zeitschrift*, 14(5/6): 263-272.
47. Zavadskas, E. K.; Antuchevičienė, J. (2006). Development of an indicator model and ranking of sustainable revitalization alternatives of derelict property: A Lithuanian case study. In: *Sustainable Development*, 14(5): 287-299.
48. Zavadskas, E. K.; Zakarevičius, A.; Antuchevičienė, J. (2006). Evaluation of ranking accuracy in multi-criteria decisions. In: *Informatica*, 17(4): 601-618.

DIDKREČIŲJŲ ALTERNATYVŲ DAUGIATIKSLIS OPTIMIZAVIMAS SANTYKIŲ ANALIZĖS PAGRINDU

Willem Karel M. Brauers
Antverpeno universitetas,
Belgija

Edmundas Kazimieras Zavadskas
Vilniaus Gedimino technikos universitetas, Lietuva

Remiantis santykių analize buvo suformuoti du metodai, skirti daugiatiksliam optimizavimui su diskrečiomis alternatyvomis. Pirmasis metodas yra MOORA (daugiatikslis optimizavimas remiantis santykių analize), kuris veikia kaip atsakymų matrica tikslų alternatyvoms, kurioms yra taikomi santykiai. Šie santykiai laikomi geriausiu pasirinkimu iš skirtingų santykių pavyzdžių. Galutiniai rezultatai, kintantys nuo nulio iki vieneto, yra sudedami arba atimami norint sumažinti tikslų skaičių. Galiausiai visos alternatyvos yra surikiuojamos į prioritetų eilę pagal gautų bemačių skaičių dydį. Geriausias būdas tikslui suteikti didesnį reikšmingumą yra šį tikslą pakeisti papildomais tikslais. Antrasis metodas yra atskaitos taško metodas. Jis paremtas ta pačia argumentacija kaip ir MOORA. Atskaitos taško metodas yra taikomas pirmosios MOORA dalies rezultatams patikrinti.

Willem K. M. Brauers holds the following degrees: Doctor of Philosophy in Economics (University of Leuven), Master of Arts in Economics (Columbia University, New York), Master of Arts in Management and Financial Sciences, Master of Arts in Political and Diplomatic Sciences and Bachelor of Philosophy (University of Leuven). He is a professor at the Faculty of Applied Economics and at the Institute for Development Policy and Management of the University of Antwerp. He is a former professor at the University of Leuven, the Belgian War College, the School of Military Administrators, and the Antwerp Business School. He was a research fellow in several American institutions, such as the Rand Corporation, the Pentagon, the Institute for the Future, the Futures Group and extraordinary advisor to the Center for Economic Studies of the University of Leuven. He was a consultant in the Belgian Department of National Defence, the Department of Industry in Thailand, the project for the construction of a new port in Algeria (the port of Arzew), in the international seaport of Antwerp and, generally, he was a consultant for electrical work. He was the Chairman of the Board of Directors of SORCA Ltd. Brussels, Management Consultants for Developing Countries, associated with the group of ARCADIS. Presently he is the Chairman of the Board of Directors of MARESCO Ltd. Antwerp. He is a member of many international scientific organizations. Research interests: optimizing techniques with several objectives, forecasting techniques, public sector economics (such as for national defence and for regional sub-optimization), input-output techniques.

Edmundas Kazimieras Zavadskas holds a Sc.D. degree, is a professor, Doctor Honoris Causa of the universities of Poznan, Saint Petersburg and Kiev, vice rector of Vilnius Gediminas Technical University (Lithuania). He is a member of the Lithuanian Academy of Sciences, the president of the Lithuanian Operational Research Society, the president of the Alliance of Experts of Projects and Building of Lithuania. He is the editor-in-chief of the journals: *Journal of Civil Engineering and Management, Technological and Economic Development of Economy;* an editor of the *International Journal of Strategic Property Management.* In 1973 he was awarded a Ph.D. degree in building structures. He worked as an assistant, senior assistant, associate professor, professor at the Department of Construction Technology and Management. In 1987 he was awarded a Sc.D. degree at Moscow Civil Engineering Institute (construction technology and management). He is an author of 14 monographs in Lithuanian, English, German and Russian. Research interests: building technology and management, decision-making theory, automation in design, expert decision support systems.

Willem Karel M. Brauers yra gavęs šiuos mokslo laipsnius: ekonomikos mokslų daktaro (Leuveno universitetas), ekonomikos magistro (Kolumbijos universitetas, Niujorkas), vadybos ir finansų bei politikos ir diplomatijos mokslų magistro

(Leuveno universitetas). Jis yra Antverpeno universiteto Taikomosios ekonomikos fakulteto ir Politikos plėtros bei vadybos instituto profesorius. Profesoriaus pareigas ėjo Leuveno universitete, Belgijos karo koledže ir Antverpeno vadybos mokykloje. Jis bendradarbiavo su keletu Amerikos institutų bei buvo ypatinguoju Lueveno universiteto Ekonomikos mokslų centro patarėju, Belgijos krašto apsaugos ministerijos bei Pramonės departamento Tailande, naujo uosto Alžyre statybos projekto, taip pats tarptautinio jūrų uosto Antverpene statybų konsultantu. Buvo akcinės bendrovės SORCA Ltd. Briuselyje direktorių tarybos pirmininku bei vadybos konsultantu besivystančioms šalims. Šiuo metu jis yra akcinės bendrovės MARESCO Ltd. Antverpene direktorių tarybos pirmininkas. W. K. M. Brauers yra daugelio tarptautinio mokslo organizacijų narys. Mokslo interesų sritys: kelių tikslų optimizavimo metodika, prognozavimo metodika, viešojo sektoriaus ekonomika.

Edmundas Kazimieras Zavadskas yra habilituotas mokslų daktaras, Poznanės, Sankt Peterburgo ir Kijevo universitetų garbės daktaras, Vilniaus Gedimino technikos universiteto pirmasis prorektorius. Taip pat jis yra Lietuvos mokslų akademijos narys korespondentas, Lietuvos operacijų tyrimo asociacijos ir Lietuvos statinių ir projektų ekspertų sąjungos prezidentas, trijų žurnalų *Journal of Civil Engineering and Management, Technological and Economic Development of Economy, International Journal of Strategic Property Management* vyriausiasis redaktorius. 1973 m. jam suteiktas daktaro laipsnis statybos konstrukcijų srityje. Taip pat ėjo asistento, vyr. asistento, docento bei profesoriaus pareigas Statybos technologijų ir vadybos katedroje. 1987 m. Maskvoje, Statybos inžinerijos institute, jam suteiktas habilituoto daktaro laipsnis. Autorius yra parašęs 14 monografijų lietuvių, anglų, vokiečių ir rusų kalbomis. Mokslinių interesų sritys: pastatų technologija ir valdymas, sprendimų priėmimo teorija, projektavimo automatizavimas, ekspertinės sprendimų paramos sistemos.