



## ASSESSING REASONABLE LIMITS OF INEQUALITY ON THE BASIS OF A NEW MODEL OF INCOME DISTRIBUTION

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**Abstract.** In the first section of the work, problems of inequality in Russia are examined. In the second section, the new model of income distribution developed by author and its application for analysis of the inequality problems are discussed. In the core of the method is the assumption that one may construct a specific sequence based on the polynomial with all its roots on the unit circle and characterized by some parameter determined as an income indicator. The work makes several contributions to the basic theory of measurement and analysis of inequality. It is shown using specially constructed utility function, calculations of the Shapley value, and analysis of some dynamic models that at present the preferable level of inequality for developed countries corresponds to the value of Gini coefficient in the range approximately from 0.25 to 0.40. The results obtained show that problems of ethics are now the most important for sustainable economic growth.

**JEL classification:** D310, E640, C420.

**Keywords:** encone distribution, limits of inequality, measurement of inequality.

**Reikšminiai žodžiai:** pajamų pasiskirstymas, nelygybės ribos, nelygybės išmatavimas.

### Introduction

At present, the problem of income inequality growth is urgent in many countries. It is very serious in countries with transition economy. Inequality is growing in Russia as well. Therefore, the Russian government has taken definite measures for increasing the minimum wage to the level that corresponds to the living minimum. A special attention is paid for increasing the real size of pensions.

The growth of inequality is typical also of the richest country—the United States—where the income inequality began to grow steadily with transition to a system of floating exchange rates (Gini coefficient increased from 0.394 to 0.462 in 1970–2000). Similar trends are typical in the dynamics of income inequality in the most developed countries [Weinberg]. An analysis shows that the growth of income and wealth inequality begins in the periods of transition from economic system with strong restrictions on some parameters (the gold standard system, system of centralized planning, etc.) to liber-

alized system (breaking down the post-war international gold standard, transition to market economy, globalization with high mobility of capital and new financial instruments, etc.).

In this connection it is interesting to consider experiments with artificial society. Thus, in [J. Epstein and R. Axtell (1995)] the attempt to simulate economic life with help of computer was made with very simplistic basic set of rules of the game. The model included people with a few basic abilities, and an environment with some natural resources. It was used to set off a chain reaction of economic activity.

The authors created an imaginary sugar island named Sugarscape where the only one resource was sugar, and each square in the grid of the island had different amounts of sugar piled on it. The amounts of the sugar were different, and the sugar piles were arranged such that there were two sugar mountains separated by an area with little or no sugar. Each virtual person (agent) on Sugarscape could only search for sugar, move, and eat sugar, and had a metabolism for consuming the resource.

The game begins with chaotic movements of the agents looking for sugar but then an order begins to emerge. At the beginning of the simulation, a fairly egalitarian society existed with the smooth distribution of wealth but after several stages of the game the gap between the richest and the poorest agents increased significantly, and the distribution of wealth became very skewed. The graphs given in the book allowed us to obtain the following approximate estimates: even after the first stage of the activity of the agents the ratio of wealth of the 20% (approximately) rich and the poor group has reached 3.9 times and after the second stage 14.1 times, and the ratio of wealth of the 30% (approximately) rich and the poor group has reached 2.9 times after the first stage and 9.8 times after the third stage.

Thus, it was shown that the skewed distribution of a typical property in the society of the agents lead to very different outcomes for them. The authors explained their estimates by the influence of combination of all factors (the shape of the physical landscape, the genetic endowments of the agents, the rules that they follow, the dynamics of their interactions with each other and with their environment, etc.).

The result of the simulation proves the existence of the horizontal inequality (when people of similar origin, intelligence, and other qualities do not have equal success and have different status, income, wealth) though the traditional economic theory predicts that horizontal inequality should not exist in a free market [Beinhocker, 2006].

We see almost the same phenomenon, as in very primitive artificial society, not only in transitional economies (for example, in Russia and republics of the former USSR) but even in the USA where, as W. Sundstrom [Sundstrom, 2003] noted: “the tremendous concentration of income and wealth at the top of the income distribution poses a number of ethical challenges as well. For instance, in a society in which money buys political power and access, we must fear and actively resist the rise of an entrenched plutocracy”.

The globalization and present world financial system stimulate increasing income inequality. A significant complication of new knowledge, the desire to get rich quickly, lack of social responsibility, etc. lead to increasing risks of different kind (for example risks connected with food safety, see [Varshavsky, 2009]). Many problems of the modern life stem from moral, political, and economic errors. That is why the solutions have to be found in the moral, economic, and political fields. Thus, Ch. Clark writes: “New approaches and policies that promote both equity and efficiency, and not trade one off the other, are needed” (Clark Ch., 2002).

The central idea of this interdisciplinary work (the study examines joint problems of economics, sociology, mathematics, ethics) is to show that the optimal level of inequality exists. The author developed the new model of income distribution and applied it to the analysis of the inequality problems. The results obtained show that problems of ethics are now the most important for sustainable economic growth.

#### Growth of income inequality in Russia

Income differentiation of the population of Russia increased steadily since 1991. According to the data of Rosstat, in 2006 the 10% of the most rich received 30.2% of the total income (in 2005, 29.9%), and the 10% with lowest income got only 2%. Those with income upto 4500 rubles obtained 25.5%, less than 6000 rubles—39%, from 6000 to 12000 rubles—34.6%, and more than 12000 rubles—26.4% of the total population income. One should take into account that the data of Rosstat on income differentiation are understated. According to the data from the specific surveys and the World Bank, the actual level of inequality is considerably higher (table 1) (Rimashevskaya, 2005), (Shevyakov&Kiruta, 2002), (Shevyakov, 2005a, b), (Aivazyan, 1997), (Varshavsky, 2007a, b).

**Table 1.** Income distribution in Russia by 20 percent groups (100% = total income; the 1st group has the lowest income, the 5th group corresponds to the top 20 percent), %

No of the 20 percent group	Data of Rosstat 1)							RLMS2)	RLMS	RLMS
	1970	1980	1990	1995	2000	2005	2007	1998	2000	2004
1	7.8	10.1	9.8	6.1	5.9	5.5	5,1	3.9	3.9	3.7
2	14.8	14.8	14.9	10.8	10.4	10.2	9,8	8.0	7.7	8.8
3	18	18.6	18.8	15.2	15.1	15.2	14,8	12.7	12.7	12.8
4	22.6	23.1	23.8	21.6	21.9	22.7	22,5	20.7	21.3	21.9
5	36.8	33.4	32.7	46.3	46.7	46.4	47,8	54.7	54.3	52.7
5:1ratio	4.72	3.31	3.34	7.59	7.92	9.37	14.03	13.92	14.24	
Gini coefficient (Rosstat)										
			0.289	0.39	0.40	0.4093) 0.5674)	0.422			
Gini coefficient (data from other sources)										
			0.2385)		0.4536)	0.4536)		0.4805)		

1) 1970-1990 - total income (including the cost of net output of the household activity of population). 2) RLMS - Russian Longitudinal Monitoring Survey of the economy and health (Besstremyanaya, 2007). 3) Russia. 4) Moscow. 5) Data for 1988 and 1998 (Milanovic, 1998). 6) World Income Inequality Database (WIID) data.

The level of the income inequality of the population of Russia, estimated by the Gini coefficient, considerably exceeds the same index for the countries of Eastern Europe, where the transformation of economic system have begun earlier; it is also higher than in the states of the CIS (table 2).

**Table 2.** Inequality of earnings and income in transition economies (Gini index)

Country	Inequality of the earning distribution *	Year	Income inequality **	Year
Czech Republic	27.3	2002	23.5/25.4	2004
Slovenia	30.3	2004	24.3/28.4	2003
Latvia	32.1	2004	39.1/33.6	2004
Belarus'	33.8	2004	24.8/31.9	2004
Poland	35.1	2004	36.6/34.1	2004
Rumania	35.8	2003	35.9/30.3	2004
Kazakhstan	37.0	2004	.../29.1	
Hungary	38.6	2001	26.8/26.9	2003
Estonia	38.8	2001	40.2/37.2	2003
Lithuania	39.4	2004	30.9/31.9	2004
Ukraine	41.0	2004	32.7/32.0	2002
Russia	46.9	2004	42.2/40.7	2001
Russia	-		-/45.6	2005
Moscow	-		-/56.7	2005

\* data of TransMONEE 2006 Database.

\*\* Numerator - data of TransMONEE 2006 Database; denominator - data of the World Bank (for Russia, Belarus, Kazakhstan and Ukraine - data of Rosstat).

A special problem is the low level of pensions, gradually falling relatively to an average wages. The relationship of average pensions and wages in the period of reformation of economic system was reduced from 33.7% in 1990 to 22.8% in 2007 (25.6% in 2006), and for skilled workers—to 15–20%. The flat rate of the income tax equal to 13%, practically do not create basis for redistribution and contributes to the conservation of high share of the poor population.

According to the results of the survey conducted recently by All-Russian Center for studies of the public opinion (VCIOM), at present, 83% of respondents do not attribute themselves to the middle class (VCIOM, 2008).

A special attention should be focused on the serious stratification of population in Moscow, where the Gini index was equal to 56.7 in 2005 and 62.7 in 2002, which is higher than in Nigeria (50.6) and

close to the Gini index of Brazil (58.5), Republic of South Africa (59.3), Botswana (63.0), Central African Republic (61.3), Swaziland (62.9), Sierra Leone (62.9) (World Development Indicators, 2004).

The situation in Russia was very strictly characterized by Joseph Stiglitz, winner of the 2001 Nobel Prize for Economics. He wrote on the results of economic transitions in Russia as follows:

“For the majority of those living in the former Soviet Union, economic life under capitalism has been even worse than the old Communist leaders had said it would be... The middle class has been devastated, a system of crony and mafia capitalism has been created, and the one achievement, the creation of a democracy... appears fragile at best... While those in Russia must bear much of the blame for what has happened, the Western advisers, especially from the United States and the IMF, who marched in so quickly to preach the gospel of the market economy, must also take some blame. At the very least, they provided support to those who led Russia and many of the other economies down the paths they followed, arguing for a new religion — market fundamentalism—as a substitute for the old one—Marxism”(Stiglitz, 2002, pp. 133–134). “Privatization, accompanied by the opening of the capital markets, led not to wealth creation but to asset stripping.” (Stiglitz, 2002, p. 144). “It was expected that Russia would be spared the inequality arising from inherited wealth. Without this legacy of inherited inequality, there was the promise of a more egalitarian market economy. How differently matters have turned out! Russia today has a level of inequality comparable with the worst in the world, those Latin American societies which were based on a semi-feudal heritage. ...And the prognosis for the future is bleak: extremes of inequality impede growth, particularly when they lead to social and political instability” (Stiglitz, 2002, pp. 154–155).

The situation with inverted scale of values, when the high-skilled workers are undervalued, that took place in Russia after 1991, presents a special danger. Thus, in Moscow the average wages in the Research and Development (R&D) sector is only 3.2 times and in education 2.7 times higher than the living minimum. At the same time in some branches of the Moscow economy, the wages are more than two times higher than in the R&D sector and education (table 3). This situation does not give stimuli for improvement of the level of qualification and knowledge and leads to significant reduction in the inflow of young people into the R&D and other branches of the knowledge-based sectors.

The concentration of income and wealth in the narrow stratum of the Russian society stimulates the outflow of savings abroad and excessive expenditures for the foreign objects of luxury. These proc-

esses are strengthened by the orientation towards the short term goals and uncertain future.

The low income of the major part of the population leads to worsening the population's health, delays the development of civic society, decreases the social inclusion. It does not contribute to the solidarity of population and to the growth of patriotic feelings; at the same time, it increases risks for those, who became rich after 1991. There are also some indirect indicators of strong income differentiation within the population. One of them is the serious growth of passengers on international airlines and very strong decrease on domestic ones, 3.6 and 4 times, respectively. An insufficient level of income for the major portion of the population slows down Internet access and access to information for students, scientists, and engineers; it contributes to strengthening information gap and creates the risk of significant decrease in the quality of education. In summary, the income inequality creates a negative

impact on the economic and social growth (Varshavsky, 2007a, b).

Thus, development occurs in accordance with the Darwinian law: "It is not the strongest of the species that survive, nor the most intelligent, but the ones most responsive to change" (Charles Darwin, 1856).

After the end of primary privatization, the increase in the income inequality is influenced by the low level of wages in the majority of industries and simultaneously very high payments in some branches. The degree of the underestimation of workers can be seen by comparing the average values of annual wages to the Gross Domestic Product (GDP) per capita ratio for Russia and other countries.

**Table 3.** Average monthly wages in Moscow, 2005–2009 (estimated by the data of Mosgorstat)

Indicator	Average monthly wages, rubles				As a percentage of the financial organizations wages			
	1.12.05	1.1.07	1.1.08	1.1.09	1.12.05	1.1.07	1.1.08	1.1.09
Average monthly wages of the workers	18941	31872	42170	50068	0.54	0.48	0.57	0.51
Financial organizations	35248	66729	73440	98464	1	1	1	1
Telecommunications	24291	34871	55261	51968	0.69	0.52	0.75	0.53
Wholesale and retail trade	20827	31038	38601	43554	0.59	0.47	0.53	0.44
R&D	...	28330	37458	46711	...	0.42	0.51	0.47
Construction	18190	29726	39168	42851	0.52	0.45	0.53	0.44
Health and social services	13737	24127	31980	42549	0.39	0.36	0.44	0.43
Education	13019	21456	28181	38545	0.37	0.32	0.38	0.39
Manufacturing	18657	23334	34395	35036	0.53	0.35	0.47	0.36

In total, this index for Russia is approximately 65% of the USA level. It is the highest in Russia for financial services (almost the same, as in the USA: 98.1%), accommodation and food services (95.2%), transport and telecommunications (79.4%). It is the lowest in education (41.7%), agriculture (45.7%), public health (51.3%), manufacturing (54.4%), and R&D sector (60.6%) (table 4).

1) For the USA—telecommunications, information and data processing services.

These data show, on one side, that wages in Russia are approximately 2 times lower, and, on the other side, that the high inequality of payment by branches exists (mostly for the branches, which determine the development of the knowledge-based society (Makarov V.L., Varshavsky A.E., 2004)).

These data also show that wages in Russia could be increased approximately 1.5 times overall and 2 times and even more in the manufacturing and knowledge sector. This conclusion coincides with

**Table 4.** Average annual wages to the GDP per capita ratio by industry, % (estimated using data of Rosstat, 2007, and BEA USA 2004)

	Russia	USA	Russia's level as a percentage to the USA level, %
Finance and insurance	180.1	183.6	98.1
Accommodation and food services	47.7	50.1	95.3
Real estate	86.9	103.4	84.0
Petroleum and coal products	157.1	191.3	82.1
Transportation	84.9	106.9	79.5
Mining, except oil and gas	98.9	135.8	72.8
Construction	75.1	105.5	71.2
Primary metals	91.0	130	70.0
Retail trade	45.3	70.9	63.9
Oil and gas extraction	171.8	274.7	62.5
R&D	101.2	167.1	60.6
Manufacturing	66.7	122.5	54.5
Machinery	69.9	129.7	53.9
Health care and social assistance	52.0	101.4	51.3
Agriculture, forestry, fishing	31.6	69.2	45.7
Chemical products	75.6	174.5	43.3
Education	45.3	108.6	41.7
Total	70.7	109.3	64.7

the estimation by an academician A. Aganbegyan, who proposes to double nominal wages (Aganbegyan, 2007).

The inequality of payments is also very serious problem for Russian regions. The very high differentiation of income by regions is constantly increasing because of the relatively fast growth of average income in major cities (Moscow, St. Petersburg) where it is by several times higher than in regions.

### New model of income distribution and its applications

#### *Different views of economists about inequality and economic growth*

At present, there are three different views on the impact of income inequality on economic growth (Clark Ch., 2002).

First, using the Kuznets-hypothesis (Kuznets, 1955), economic progress is initially accompanied by rising income inequality which is a contributory factor for economic growth. This point of view was supported in the middle of the second-half of the twentieth century by Okun (Okun, 1975), Kaldor (Kaldor, 1956), Mirrlees (Mirrlees, 1971), and many others. In their opinion, capital of the richest part of the population is the basic source of savings and, correspondingly, the investments which ensure economic growth. On the contrary, the lower the level of inequality the less investments into the economy; small economic stratification and (or) high level of taxation for redistributing incomes stimulate the unwillingness for the hard work and reduction in the expenditures for increasing the qualification. For example, A. Okun wrote: "We can't have our cake of market efficiency and share it equally" (Okun, 1975).

However, at the end of the twentieth and the beginning of the twenty-first century the works of Alesina, Rodric (Alesina & Rodric, 1994), Benabou (Benabou, 1996), Osberg (Osberg, 2003), Persson, Tabellini (Persson & Tabellini, 1994), Brady (Brady, 2003, 2004), Heinrich (Heinrich, 2003), Clarke R. (Clarke R., 1995), Clark Ch. (Clark Ch., 2002), and other economists have shown that an increasing share of savings of the rich part of the population decreases the aggregated consumer demand, investments, and rates of the economic growth.

The authors of the United Nations 2005 report have similar views (Human development report, 2005) where they noted that at present, the human society understands the inadmissibility of different forms of inequality (gender, ethnic, called by inherited wealth, etc.). Each person must have equal rights for education, access to the knowledge, information, to the achievements of medicine and so

forth. Adam Smith wrote in the "Wealth of Nations": "No society can surely be flourishing and happy of which the far greater part of the members is poor and miserable".

At the same time they did not deny that complete equality of the income distribution produces a significant negative effect on business and innovation activity and slows down the economic growth.

Different views of well known economists show that there is a keen need for special methodology and models elaboration that are discussed in the next section.

#### *New model of income distribution*

Currently, there are several measures of income inequality developed by Gini (Gini coefficient), Atkinson (the Atkinson index, also known as the Atkinson measure), Theil (Theil's index), Hoover (Hoover index) (Atkinson & Bourguignon, 2000). However their direct utilization for modeling and measuring the impact of inequality on economic growth is difficult.

The following new model of income distribution and its applications is proposed by the author (Varshavsky, 2003, 2007a, b, c, 2008).

**Proposition 1.** In general, the relative income (or some other resource) by groups of the population (relative to the income of the richest group which is labeled  $n$ ) can be represented as a parametric sequence, depending on some parameter  $a$ :

$$A(a, n) = \{A_1(a), \dots, A_i(a), \dots, A_n(a)\}, \quad (1)$$

$$\text{where } A_1(a) = S_1(a)/S_n(a), \dots, A_i(a) = S_i(a)/S_n(a), \dots, A_n(a) = S_n(a)/S_n(a) \equiv 1,$$

$$S_1(a) = A_1(a)/A(a), \dots, S_i(a) = A_i(a)/A(a), \dots, S_n(a) = A_n(a)/A(a), \quad (2)$$

$$A(a) = \sum_{i=1}^n A_i(a) \text{ and } \sum_{i=1}^n S_i(a) = 1,$$

$S_i(a)$  are the shares of income of the group number  $i$  in total income ( $i=1$  corresponds to the poorest group),  $a$  – parameter, defined as an inequality indicator,  $n$  – total number of equal groups of population.

It is worth to note that the functions  $S_i(a)$ ,  $i=1 \dots n$ , define points on the Lorenz curve  $L_i$ ,  $i=1, 2, \dots, n$ , as follows:

$$L_1 = S_1(a); L_2 = S_1(a) + S_2(a); \dots, L_{n-1} = \sum_{i=1}^{n-1} S_i(a); L_n = \sum_{i=1}^n S_i(a) = 1. \quad (3)$$

**Proposition 2.** The relative income shares of equal groups within the population  $A_i(a)$  in total income form a specific finite power sequence: the characteristic polynomial  $A(a)$  of the sequence has all its roots on the unit circle, where parameter  $a$  is

an inequality indicator,  $a \geq 1$  ( $a=1$  for egalitarian society).

In order to simplify the problem, we consider 20% groups of the population, i.e. distribution of income by quintiles (this restriction is not essential—we can obtain the results by some modification of the formulas for 10%, 5% etc., groups of the population, see Proposition 3 below.

One may show then that the relative income shares of 20% groups of the population in total income ( $n = 5$ ) can be approximately represented as a specific finite power sequence in which the second and penultimate terms are removed. Indeed, on the basis of empirical analysis (Varshavsky, 2003, 2007a, b, c, 2008), as well as some theoretical justification (see below) it can be shown that for the 20% income groups (quintiles) the sequence (1) is as follows (for  $n=5$ ):

$$A(a, 5) = \{A_1(a), A_2(a), A_3(a), A_4(a), A_5(a)\} = \{a^6, a^4, a^3, a^2, 1\}. \tag{4}$$

The shares of the relevant income groups in total income are determined then as follows:

$$S_i(a) = a^{-(6-i)} / A(a), \quad i=2, 3, 4; \quad S_5(a) = 1/A(a),$$

$$S_1(a) = a^{-6} / A(a); \tag{5}$$

$$A(a) = 1 + a^{-2} + a^{-3} + a^{-4} + a^{-6}; \tag{6}$$

The function  $A(a)$  is the characteristic polynomial of the sequence (4a) and one may see that all its

roots are on the circle of unit radius (some variant of cyclotomic polynomials).

A similar asymmetry in distribution was indirectly mentioned earlier by Kant and Rousseau. Rousseau singled out the most powerful and the most needy, ascribing a kind of right to property of another on the basis of their strength, or on the basis of their needs (“C'est ainsi que les plus puissants ou les plus misérables, se faisant de leur force ou de leurs besoins une sorte de droit au bien d'autrui, équivalent, selon eux, à celui de propriété, l'égalité rompue fut suivie du plus affreux désordre: c'est ainsi que les usurpations des riches, les brigandages des pauvres, les passions effrénées de tous étouffant la pitié naturelle, et la voix encore faible de la justice, rendirent les hommes avarés, ambitieux et méchants.”(Rousseau J.)).

Inequality indicator  $a$ , is, obviously, connected with the Gini coefficient. The approximate relationship between them can be obtained using the least squares method. Thus, for  $1.236 < a < 1.763$  the following estimate was obtained:

$$Gini = 0.9506 \ln(a) + 0.0535; \quad R^2 = 0.9957 \tag{7}$$

Table 5 below represents the estimated Gini coefficient as a function of inequality indicator  $a$  (7).

**Table 5.** Gini coefficient as a function of inequality indicator  $a$  (7)

$a$	1.2	1.25	1.3	1.35	1.4	1.45	1.5	1.55	1.6	1.65	1.7	1.75	1.8
Gini	0.23	0.27	0.30	0.34	0.37	0.41	0.44	0.47	0.50	0.53	0.56	0.59	0.61

The estimation of the income inequality indicator  $a$  for 39 countries on the basis of information from the database WIID on disposable monetary income (World Income Inequality Database) for 20% income groups gave the following results: the average coefficient of determination  $R^2$  (adjusted) was equal to 0.995, its minimum value was also close to 1 (min  $R^2 = 0.975$ ), see table 6 and fig. 1.

**Proposition 3.** The sequence for the 20% groups of population ( $n=5$ ) can be used as the base sequence for construction of sequences for the 10%, 5%, 2.5%, etc. income groups. All of these sequences also have all their roots on the unit circle. For example, for the 10% income groups of population one may construct on the basis of the sequence  $A(a, 5)$  the following sequence  $A(b, 10)$ , where  $b$  is the inequality indicator, see (4):

$$A(b, 10) = (C_1, C_2 b^{-2}, b^{-3}, b^{-4}, b^{-5}, \dots, b^{-8}, D_2 b^{-9}, D_1 b^{-11}). \tag{6a}$$

This sequence has to satisfy the following conditions:

- all roots of its polynomial are on the unit circle
- $(b^{-3} + b^{-4}) / (b^{-5} + b^{-6}) = a$ , or  $b^2 = a$

- $(C_1 + C_2 b^{-2}) / (b^{-3} + b^{-4}) = a^2 = b^4$  or  $C_2 b^{-2} = 1 + b - C_1$

- $(D_2 b^{-9} + D_1 b^{-11}) / (b^{-7} + b^{-8}) = a^{-2} = b^{-4}$  or  $D_2 b^2 = (1 + b) b^{-1} - D_1$ , that is two coefficients  $C_1$  or  $C_2$ , and  $D_1$  or  $D_2$  are independent

- $C_1 > C_2 b^{-2} > b^{-3}$ ;  $1 > D_2 b^{-1} > D_1 b^{-3}$

Then the characteristic polynomial for the sequence  $A(b, 10)$  is as follows:

$$A(b) = 1 + b + b^{-3} + b^{-4} + \dots + b^{-8} + (1 + b) b^{-12} \text{ or } (1 + b)(1 + b^{-4} + b^{-6} + b^{-8} + b^{-12}) = (1 + b)A(a), \quad a = b^2.$$

One may similarly construct sequences for  $n=5m, m=3, 4$ , etc.

The assessment of parameters of the sequence for the 10% income groups of population made on the basis of information from WIID on disposable monetary income for 39 countries, including Russia, also confirmed very high accuracy of approximation (we used a nonlinear estimation procedure FIML, Full Information Maximum Likelihood): the minimum value of the coefficient of determination  $R^2$  (adjusted) was equal to 0.978, and t-statistics were high., see fig. 2.

**Table 6.** Estimated values of inequality indicator *a* and real Gini coefficients for some countries.

Country	inequality indicator <i>a</i>	<i>R</i> <sup>2</sup>	Gini coefficient (WIID data)
Denmark	1.236	1.000	0.247
Slovenia	1.255	1.000	0.264
Finland	1.262	0.993	0.268
Sweden	1.265	0.998	0.272
Norway	1.267	0.988	0.274
Netherlands	1.308	0.999	0.307
Taiwan	1.322	0.991	0.319
Belgium	1.324	0.976	0.322
France	1.326	0.994	0.323
Ireland	1.348	0.994	0.341
Spain	1.354	0.999	0.345
Italy	1.372	0.999	0.358
United Kingdom	1.385	0.996	0.370
Republic of Korea	1.397	0.975	0.369
United States	1.434	1.000	0.401
Russian Federation	1.524	0.997	0.453
Argentina	1.641	0.997	0.523
Panama	1.757	0.998	0.578

**Theoretical justification for the proposed model (for the 20% income groups of population)**

The proposed model of the income distribution (4) - (6) can be justified by the following considerations (for the 20% groups of population).

Suppose that there are two mutually substitutable goods *M* and *N*, for example, income obtained within some period of time, and the time used for leisure or for work that does not bear any income to individual but gives him a pleasure, i.e. the first good is the income (or capital, wealth) obtained by an individual, and the second good is the opportunity to engage in the activity that do not bring any income, but bring a pleasure.

The process of distribution of each of these goods can be represented as follows. Assume that the good is successfully distributed among the certain number of individuals (or groups of individuals), in our case, among the five ones. A prerequisite for this distribution is a requirement of a nonzero remainder of distributed good for all participants, except the last one.

Consider first the process of successive distribution of the first good *M* - income between five participants; suppose that the total amount of this good is equal to 1. The choice is as follows: the first participant (with number *j*=5) with probability *p* chooses an income share *S* < 1 (recall that he can not take all income).

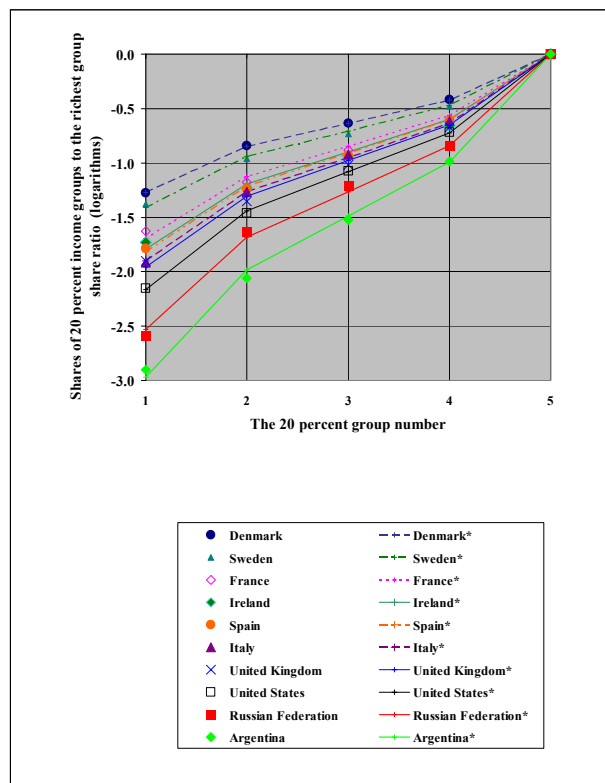
The rest of the income (1-*S*) is sequentially distributed among the other participants, which can thus claim smaller shares of the income. For the second participant with the number 4 the probability of choosing the share of income is, obviously, equal to *pq*, where *q* is the probability that he has no right to choose, *q* = 1-*p*, and a share of income selected by him is equal to *pq*(1-*S*). Similarly, the shares of income for participants with the numbers 3 and 2 are equal to *pq*<sup>2</sup>(1-*S*) and *pq*<sup>3</sup>(1-*S*).

Suppose that *S*=*p*, then we obtain the following sequence of distribution of the good *M*:

$$\{p, p(1-p)^2, p(1-p)^3, p(1-p)^4, a_1\}, \quad (8)$$

where *a*<sub>1</sub> is an income that the last participant (who has no right to choose) has received equal to the following:

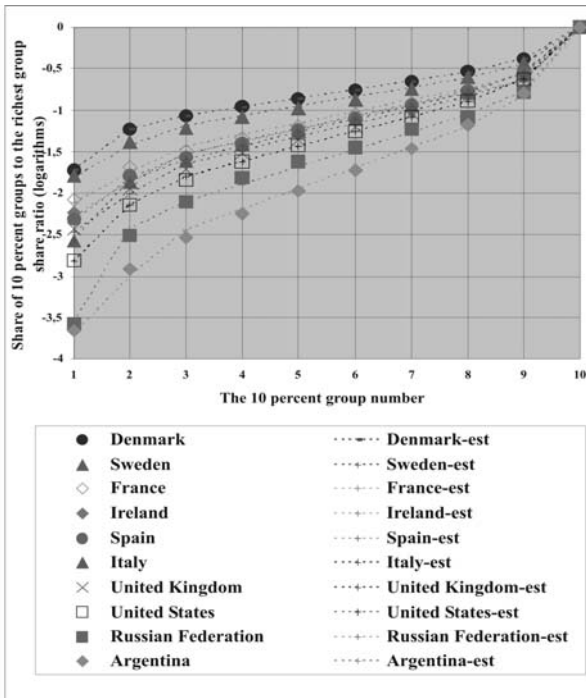
$$a_1 = 1 - [p + p(1-p)^2 + p(1-p)^3 + p(1-p)^4].$$



**Fig.1.** Real and estimated (on the basis of the proposed model) distributions of relative shares of the disposable monetary income for the 20% groups of population for some countries as a function of the 20 percent group number (points—real data, curves—results of estimation); *i*=1 for the poorest group and *i*=5 for the richest group, logarithms.

Thus, the participant with the highest number or priority (*j* = 5) has the right to choose the first part of the income. The others, with decreasing order of seniority, have the right to choose from what is left after him (we can assume that they belong to the middle class). However, they must leave some non-

zero part of the income to the participant with the lowest number ( $j = 1$ ) who has no right to choose.



**Fig. 2.** Real and estimated (on the basis of the proposed model) distributions of relative shares of the disposable monetary income for the 10% groups of population for some countries as a function of the 10 percent group number (points—real data, dashed curves—results of estimation),  $i=1$  for the poorest group and  $i=10$  for the richest group, logarithms

Next, consider the process of selecting a second good  $N$ .

Suppose that the choice of the second, intangible good (free time for pleasure) which do not give any material profit, is carried out similarly, but the right to be the first is provided to the participant number  $j = 1$ , who takes his share  $t$  of the good  $N$  with the probability  $f$ . If,  $q = 1-f$  is the probability that a participant has no right to choose and  $b_5$  is the share of the second good which remains for the participant with number  $j=5$ , and if  $t = f$  then we get the following sequence which is symmetrical to (8):

$$\{b_5, f(1-f)^4, f(1-f)^3, f(1-f)^2, f\}. \tag{8a}$$

We can assume that the second good  $N$  is selected with the same probability as the first one, i.e.  $p = f$ , and the ratio of goods chosen by each subsequent and previous participant is the same for  $i=2, 3, 4$ . Then we must have that  $p(1-p)^4/a_1=f/f(1-f)^2$  and  $p/p(1-p)^2=f(1-f)^4/b_5$ , i.e. if  $p = f$ , we obtain  $a_5=p(1-p)^6$  and  $b_5=f(1-f)^6$ . We may recall the words of the Bible: “But many who are first will be last, and many who are last will be first”:

Therefore, we can consider the following sequence of the income distribution:

$$\{p, p(1-p)^2, p(1-p)^3, p(1-p)^4, p(1-p)^6\}, \tag{9}$$

where  $p + p(1-p)^2 + p(1-p)^3 + p(1-p)^4 + p(1-p)^6 = 1$ .

We can use also the sequence that is equivalent to (9):

$$\{1, (1-p)^2, (1-p)^3, (1-p)^4, (1-p)^6\}$$

or if  $1-p = q=1/a$ , we obtain the sequence of the relative income distribution as follows:

$$A(a, 5) = \{1, a^{-2}, a^{-3}, a^{-4}, a^{-6}\} \tag{4}$$

**Determining the reasonable level of inequality**

We can use now the sequence (4a) for  $n=5$  to choose the optimum level of income inequality. Social justice does not demand that the economic pie have to be equally divided (Ch. Clark, 2002). Absolutely egalitarian society can not be stable. At the same time, economics with high level of inequality does not develop.

Based on the proposed model of income distribution, we use several methods for choosing the optimal level of inequality (Varshavsky 20032007a, b, c, 2008).

The first and the most general method is based on the analysis of interrelation between different income groups with the aid of the theory of cooperative games.

Let us limit our analysis to five groups of population with different shares of income in the total

income:  $S_1, S_2, S_3, S_4, S_5, \sum_{i=1}^5 S_i = 1$ . We use the meth-

odology given in the previous section. A share of each group  $i$  in the total income ( $i = 1$  corresponds to the poorest group and  $i = 5$  to the richest group) is equal to

$$S_i(a) = A_i(a)/A(a), \text{ see (4)-(6)}$$

**Proposition 4.** We assume that the degree of participation in the political life and the political influence of each group of population are determined by the share of its income in the total income of the country due to possibility of creating coalitions which accept the desirable political decisions. The coalition is winning if its share is equal to 50% of the total income or more.

We will use the method based on the estimation of the Shapley value. Shapley showed that there is a unique payoff function that satisfies four axioms (group rationality, symmetry, null player condition, and additivity). The Shapley value for a player is a weighted sum of his marginal contributions to all coalitions he can join. In the game with this criterion, the core consists of one distribution, in which the size of payments depends on “the force” of the player, calculated as an expected marginal contribution, which can obtain coalition when this player enters it (Shapley, 1953), (Montet&Serra, 2003), (Owen, 1982). The method is based on searching the vector of the values of game  $f(v)$ , whose dimension



is equal to  $n$ :

$$f(v) = \{f_1(v), \dots, f_n(v)\}.$$

In the case of simple game  $v$  the formula for Shapley's vector has the following form:

$$f_i(v) = \sum_T (t-1)!(n-t)!/n!,$$

where  $n$  is the number of players;  $t$  is the number of elements in  $T$ , the summing up is accomplished over all such coalitions  $T$ , which are winning. This formula expresses the Shapley value for the player  $i$  as a weighted sum of his marginal contributions to all coalitions he can join. The Shapley value evaluates the power structure in a coalition game and may be considered as index of power or index of social productivity, etc. (Montet&Serra, 2003).

The formula for  $f_i(v)$  was used for analysis of

the income distribution of the five 20 percent income groups, see (4)-(6). For every level of inequality, characterized by inequality indicator  $a$  or, correspondingly, the Gini coefficient, we have the following coalition:

$$(S_1, S_2, S_3, S_4, S_5) \text{ or } (a^{-6}/A, a^{-4}/A, a^{-3}/A, a^{-2}/A, 1/A),$$

$$\text{where } A = 1 + a^{-2} + a^{-3} + a^{-4} + a^{-6}.$$

Six variants of Shapley's vector were estimated, which correspond to six values of the inequality indicator  $a$  ( $a$  changed from 1.2 to 1.5). The shares for the chosen values of the inequality indicator ( $a=1.2$  or coefficient Gini  $\approx 0.23$ ;  $a=1.25$  and Gini  $\approx 0.27$ ;  $a=1.3$  and Gini  $\approx 0.30$ ;  $a=1.35$  and Gini  $\approx 0.34$ ,  $a=1.4$  and Gini  $\approx 0.37$ ,  $a=1.5$  and Gini  $\approx 0.44$ ), are given in table 7, see also fig.3.

**Table 7.** Shares of income of the 20 percent group in the total income and Shapley values for different levels of the income inequality (Varshavsky, 2007b, 2008)

Inequality indicator $a$	Gini coefficient (approximate values)	Shapley value					Share in the total income				
		The 20 percent income group number					The 20 percent income group number				
		5	4	3	2	1	5	4	3	2	1
1.20	0.23	0.400	0.233	0.233	0.067	0.067	0.324	0.225	0.187	0.156	0.108
1.25	0.27	0.400	0.233	0.233	0.067	0.067	0.354	0.227	0.181	0.145	0.093
1.30	0.30	0.500	0.167	0.167	0.167	0.000	0.384	0.227	0.175	0.134	0.080
1.35	0.34	0.500	0.167	0.167	0.167	0.000	0.413	0.227	0.168	0.124	0.068
1.40	0.37	0.500	0.167	0.167	0.167	0.000	0.441	0.225	0.161	0.115	0.059
1.50	0.44	0.600	0.100	0.100	0.100	0.100	0.494	0.219	0.146	0.097	0.043

The obtained results can be interpreted as follows (Varshavsky, 2007b,c).

For all  $a$  in the range from 1.3 to 1.4 (or coefficient Gini  $\approx 0.30-0.37$ ) Shapley's vector takes the same form. In this case, the component of vector, which corresponds to the fifth group (with greatest income), is equal to the sum of the Shapley's vector components that correspond to 2nd, 3rd, and 4th groups (they form the middle class), which are characterized by equal possibilities. Their political weights (or their roles in society) are identical, though their shares in total income are different; the poorest group has zero component, i.e. it has no political weight in making political decisions. To the certain degree, it is possible to assume that with such weights of all groups the system is sufficiently stable as the influence of the richest group is balanced by the middle class characterized by the joint weights of the groups 2, 3, and 4.

With the decrease in the inequality ( $a = 1.25$  or Gini  $\approx 0.27$ , and  $a = 1.20$  or Gini  $\approx 0.23$ ), the role of groups 2, 3, and 4 grows, the weight of the richest group decreases, and the weight of the poorest one becomes significant. In this case, the influence of the richest group is less than total weight not only of the all remaining groups, but also less than combined

influence of the 4th and 3rd, either of the 2nd, 3rd, and 4th, or of the 4th, 3<sup>rd</sup>, and 1st groups, but together with the 2nd and 1st groups the weight of the 5th group becomes sufficient, i.e. there are several variants of payoff that indicates on the insufficient stability of the society.

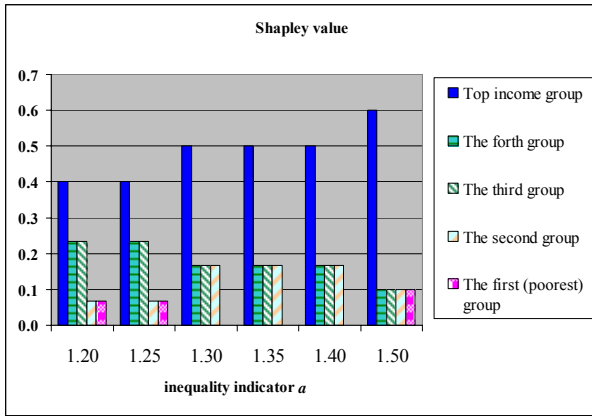
When the inequality increases ( $a = 1.5$  or coefficient Gini  $\approx 0.4$ ), the fifth group begins to drive a situation as the sum of weights of the rest of the groups, though they are equal, already cannot impact on the decision making, see fig. 3 (Varshavsky, 2007b, 2008).

The second method of choosing the optimal level of inequality is based on using a special utility function as follows:  $U = a[(1a^{-2}a^{-3}a^{-4})/(1+a^{-2}+a^{-3}+a^{-4}+a^{-6})^5]^{1/5}$ . This method gives the optimal value of the inequality indicator in the same range: it is equal to  $a=1.30$  (Gini coefficient equal approximately to 0.30) (Varshavsky, 2007a,b).

Using the proposed model of income distribution for the 20% income groups we can conclude the following:

1. significant inequality characterized by the coefficient Gini more than 0.40-0.45 (as is in the USA, Russia, etc.) must negatively affect the economic growth in the long run,

2. low level of inequality leads to unstable society.



**Fig. 3.** Shapley vector for different levels of income inequality: 1) inequality indicator  $a=1.2$ , coefficient Gini $\approx 0.23$ ; 2)  $a=1.25$ , Gini $\approx 0.27$ ; 3)  $a=1.3$ , Gini $\approx 0.30$ ; 4)  $a=1.35$ , Gini $\approx 0.34$ ; 5)  $a=1.4$ , Gini $\approx 0.37$ ; 6)  $a=1.5$ , Gini $\approx 0.44$  (Varshavsky, 2007b,c).

1. To prove the first, we use the following modification of the Keynes's model. Assume that consumption of each income group is proportional to the disposable income  $Y_d$ :  $C_i = C_{li} Y_d$ ,  $i=1, \dots, n$ , with different coefficients  $C_{li}$ . These coefficients depend both on the level of inequality and the level of savings which is minimal for the group with the lowest income and maximum for the one with the highest income. The total volume of consumption  $C$  can be represented as the sum of the volumes of consumption  $C_i$  for all groups,  $i = 1, \dots, n$ :

$$C = \sum_{i=1}^n C_i = Y_d \sum_{i=1}^n C_{li}$$

This hypothesis corresponds with the assumption of Kaldor (Kaldor, 1956) that the propensity to save is different for capitalists and workers (see also Pasinetti, 1962). Similarly to the idea of Kaldor, we can assume that propensity to save for the richer population group is considerably higher than for poorer groups and, correspondingly, propensity to consume is relatively lower. The propensity to consume  $f$  of each income group depends on the inequality indicator  $a$  and the number  $i$  of the group:

$f = f_i(a)$ , where  $f_1(a) > f_2(a) > \dots > f_n(a)$ ,  $a$  - inequality indicator

Thus, the total function of consumption can be defined as

$$C = C_{10} Y_d [A_1(a) f_1(a) + A_2(a) f_2(a) + \dots + A_n(a) f_n(a)]$$

For  $n=5$  we obtain as follows:

$$C = C_{10} Y_d [f_1(a) a^{-6} + f_2(a) a^{-4} + f_3(a) a^{-3} + f_4(a) a^{-2} + f_5(a)] / A(a) = Y_d K(a, f),$$

where  $A(a) = a^{-6} + a^{-4} + a^{-3} + a^{-2} + 1$ ,

$$\text{and } K(a, f) = \frac{C_{10} [f_1(a) a^{-6} + f_2(a) a^{-4} + f_3(a) a^{-3} + f_4(a) a^{-2} + f_5(a)]}{A(a)} \tag{10}$$

One of the possible alternatives for function  $f$  can be chosen as follows:

$$f_i(a) = e^{-q(a-1)^i}, \quad i=2, 3, 4; \quad f_1(a) = 1, \quad f_5(a) = e^{-q(a-1)^6},$$

where  $a > 1, 0 < q < 1$ .

Obviously, the total consumption  $C$  becomes lower with growth of the inequality indicator  $a$ :  $dC/da < 0$ , since  $dK/da < 0$ , and respectively, the value of the multiplier  $\mu$  also decreases:

$$\mu = 1 / (1 - K(a, f)).$$

For Russia, as it was estimated in Varshavsky & Kosmacheva, 2003,  $\mu \approx 1.9$ . Using this value of multiplier for modeling we have shown for example, for  $q=0.05$  that increasing inequality from the level determined by Gini coefficient  $\approx 0.30$  ( $a=1.3$ ) to the level with Gini  $\approx 0.50$  ( $a=1.6$ ) leads to decreasing the multiplier by 8.2%. Correspondingly, the effect of additional investment or governmental expenditures on the economic growth will be lower.

2. To show that egalitarian society is unstable, we consider the dynamic properties of the characteristic polynomial  $A(a)$ .

If the total income (or other resources) of the 20% groups at time  $t$  is equal to  $g_t$ , then we have  $x_t + a^{-2} x_{t-2} + a^{-3} x_{t-3} + a^{-4} x_{t-4} + a^{-6} x_{t-6} = g_t$ , see (4)-(6), that is we should analyse the difference equation

$$x_t = -a^{-2} x_{t-2} - a^{-3} x_{t-3} - a^{-4} x_{t-4} - a^{-6} x_{t-6} + g_t \tag{11}$$

Dynamic properties of (11) depend, on the inequality indicator  $a$ . Using the discrete Z-transform, we obtain the expression in the Z-plane (Z-transform converts a discrete time-domain signal, which is a sequence of real numbers, into a complex frequency-domain representation [see, for example, (Jury, 1958), Varshavsky, 1984]):

$$X(Z) = G(z) / [1 + (az)^{-2} + (az)^{-3} + (az)^{-4} + (az)^{-6}] = G(z) F(a, 5, z),$$

where  $X(Z)$  and  $G(Z)$  - are Z-transforms of variables  $x_t$  and  $g_t$ .

Polynomial  $F(a, 5, z)$  can be represented as follows:

$$F(z, 5, a) \equiv F(z, a) = F_1(z, a) [1 - (az)^{-5}] / [1 - (az)^{-1}] = F_1(z, a) \{ [1 - b(az)^{-1} + (az)^{-2}] [1 - c(az)^{-1} + (az)^{-2}] \},$$

where  $F_1(z, a) = 1 - (az)^{-1} + (az)^{-2}$ .

Coefficients  $b$  and  $c$  are real and satisfy the following conditions:  $bc = -1$  and  $b + c = -1$ , i.e. coefficients  $b$  and  $c$  obey the law of the Golden Rule:

$$b = (-1 + \sqrt{5}) / 2 = 0,618; \quad c = (-1 - \sqrt{5}) / 2 = -1,618.$$

Possibly, this property to a certain extent may be used as an additional proof of existence of the proposed income distribution law described by the sequence (6) (one may remember the ethic of reciprocity, more commonly known as the Golden Rule or an ethical code that states one has a right to just treatment, and a responsibility to ensure justice for others; as stated in the Bible: "Do unto others as you would have them do unto you").

To analyze the dynamic properties of the polynomial  $F(z, a)$  we can use bilinear transform:

$$w = (z-1)/(z+1), z = (1+w)/(1-w).$$

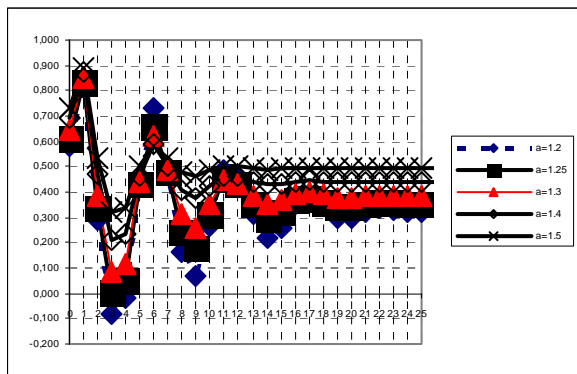
Then the expression  $[1-d(az)^{-1}+(az)^{-2}]$  will be in the  $W$ -plane as follows:

$$(a^2-ad+1)[T^2w^2+2\xi Tw+1]/[a^2(1+w)^2],$$

where  $T^2=(a^2+ad+1)/(a^2-ad+1)$ ,  $2\xi T = (a^2-1)/(a^2-ad+1)$  and  $d=b$  or  $c$ .

The damping ratio  $\xi$  that determines the level of oscillatory processes is equal to the following:  $\xi=(a^2-1)/[(a^2+1)^2-(ad)^2]^{0.5}$ . The last expression shows that the damping ratio decreases, i.e. the system becomes less stable when the inequality indicator  $a$  decreases. For  $a = 1$  (an egalitarian society) we obtain  $\xi=0$  that corresponds to an oscillatory process.

The simulation of transition processes of the redistribution of resources among the 20% income groups shows decreasing power of the richest group relatively to the other groups with transition to egalitarian society. At the same time, the results of simulation with the help of the lag model (11) show that with an increasing economic inequality system becomes more stable (Fig. 4); thus, for  $a > 1.4$  the probability of losing power by the richest group is close to zero (see also the corresponding estimates of the vectors Shapley).



**Fig.4.** Transition processes of formation of the share of income of the richest 20% group at different levels of inequality (before  $t = 0$  the incomes of all 20% groups were equal to 0.2; the income increases by 1 at  $t = 0$ ). Equilibrium values of the share of the richest group are as follows: 32.4% for  $a = 1,2$ ; 35.4% ( $a = 1,25$ ); 38,4% ( $a = 1,3$ ); 44,1 % ( $a = 1,4$ ) and 49,4% ( $a = 1,5$ ).

Thus we can formulate the main proposition.

**Proposition 5.** At present, the preferable level of inequality for developed countries corresponds to the value of the inequality indicator  $a$  in the range of  $1.25 < a < 1.45$ , or to the coefficient Gini approximately between 0.25 and 0.40. The higher level of income inequality leads to decreasing the economic growth and the lower level of inequality can result in unstable society, aggravating political, social and

economic problems. The limits of change found for the inequality indicator  $a$  and Gini coefficient correspond to the best conditions for the development of the middle class. If the indicators are outside these limits, we can talk about the aggravation of problems related to ethics. This conclusion is proven also by the indicators typical for the most developed European nations (Gini coefficient between 0.24 and 0.36, see table 6).

### Conclusions

The data from our study shows that the significant income inequality leads to the lower rates of economic development, hampers transition to the civic society. Thus, excessive stratification (when the Gini coefficient is higher than 0.45–0.50) decreases the rates of economic growth and level of democratization, whereas the weak inequality (the Gini coefficient below 0.20–0.25) leads to significant instability in society. These results show that there is some optimal level of inequality.

Resolvment of the universal ethics problems becomes the most urgent task. Human society must understand that economic development must subordinate to the objectives of a higher order. There is a keen need for revision of the principles of liberal capitalism focused mainly on maximization of profit without restraints upon people’s aspirations and appetites, when just competition becomes impossible.

New ethics is necessary for normal functioning of the society, its development, economic, social, cultural, moral, and political well-being. We need to balance between responsibility and freedom, between market laws and public social responsibility. The wealth provides not only ample opportunities, but also imposes great responsibility on the person who possesses it. We have to remind the words of A. Sen that “the ethics related view of motivation and of social achievement must find an important place in modern economics” [Sen, 1987].

### References

1. Aganbegyan A. (2007). Wages and social reforms - whether we go with them to there? The economic strategy, N 5-6/2007, in Russian.
2. Aivazyan S. (1997). Model of formation of population distribution in Russia by the value of income per capita. Economics and Mathematical Methods, vol 33, no. 4, in Russian.
3. Alesina, A. and Rodrik, D. (1994). “Distributive Politics and Economic Growth.” Quarterly Journal of Economics 109:465-90.
4. Atkinson A.B., Bourguignon F. (2000) (Eds). Handbook of Income Distribution. Volume I, North Holland.

5. Beinhocker, Eric. *The Origin of Wealth: Evolution, Complexity, and the Radical Remaking of Economics*. Harvard Business School Press, 2006.
6. Benabou R. Inequality and Growth (1996), LIS/LES Working Papers No. 142. June.
7. Besstremyannaya G. (2007). Economic analysis of healthcare system in Russia. – Moscow, CEMI Russian Academy of Sciences, in Russian.
8. Brady D. (2003). The Poverty of Liberal Economics, LIS/LES Working Papers, No. 343, March.
9. Brady D. (2004). The Welfare State and Relative Poverty in Rich Western Democracies, 1967-1997, LIS/LES Working Papers, No. 390, October.
10. Clark Ch.M.A.(2002). The basic Income guarantee. The Lify Press, Dublin.
11. Clarke, R. (1995). "More Evidence on Income Distribution and Growth," *Journal of Development Economics* 47, 403-427.
12. Epstein J.M. and Astell R., 1966. *Growing Artificial Societies*. Brookings Institution Press and MIT Press 1996 c. 228pp.
13. Heinrich G.( 2003). More is Not Necessarily Better: An Empirical Analysis of the Inequality-Growth Tradeoff Using the Luxembourg Income Study, LIS/LES Working Papers No.344., March.
14. Human development report. UN, 2005.
15. Jury E., *Sampled-Data Control Systems*, John Wiley & Sons, 1958.
16. Kaldor, N. (1956). "Alternative Theories of Distribution." *Review of Economic Studies*. 23:83-100.
17. Kuznets, S. (1955). Economic Growth and Income Inequality. *American Economic Review* 45, 1-28.
18. Makarov V.L., Varshavsky A.E., eds. (2004). "Innovation Management in Russia (problems of strategy and science and technology security)". Moscow, Nauka, monograph, in Russian.
19. Milanovic, Branko. Income, inequality, and poverty during the transition from planned to market economy, *The International Bank for Reconstruction and Development // The World Bank*, 1998.
20. Mirrlees, J.A. (1971). "An Exploration into the Theory of Optimum Income Taxation". *Review of Economic Studies* 38:175-208.
21. Montet C., Serra D. (2003). *Game theory and economics*. Palgrave Macmillan.
22. Okun, Arthur M. (1975). *Equality and Efficiency: The Big Tradeoff*. Washington, DC:Brookings Institution.
23. Osberg L., Schwabish J., Smeeding T. (2004) *Income Distribution and Social Expenditures: A Crossnational Perspective* (2004), LIS/LES Working Papers No. 350. May 2003 (revised October 2004).
24. Owen G. *Game theory* (1982). Academic Press Inc., Orlando, Florida.
25. Pasinetti L. (1962). Rate of Profit and Income Distribution in Relation to the rate of Economic Growth. *The Review of Economic Studies*. Vol. 29, Issue 4 (Oct.), 267-279.
26. Persson, T., Tabellini G. (1994). "Is Inequality Harmful for Growth?" *American Economic Review* 84:600-21.
27. Rimashevskaya N. (2005). On problem of decreasing poverty and inequality. "Economic Science of modern Russia", № 3 (30), in Russian.
28. Rousseau J. *Discours sur l'Origine et les Fondements de l'Inégalité parmi les Hommes*. [http://un2sg4.unige.ch/athena/rousseau/jjr\\_ineg.html#Note%2017](http://un2sg4.unige.ch/athena/rousseau/jjr_ineg.html#Note%2017)
29. Scruggs L., Allen J. (2005). *The Material Consequences of Welfare States: Benefit Generosity and Absolute Poverty in 16 OECD Countries*. LIS/LES Working Papers No. 409. April.
30. Sen A. *On Ethics and Economics*. Oxford, Blackwell Publishing, 1987
31. Shapley L. (1953). A Value for n-person Games. In *Contributions to the Theory of Games*, volume II, by H.W. Kuhn and A.W. Tucker, editors. *Annals of Mathematical Studies*, v. 28, pp. 307-317. Princeton University Press.
32. Shevyakov A. (2005). Social inequality, poverty and economic growth. "Society and Economy". № 3, in Russian
33. Shevyakov A. (2005). Social policy and distributive relations: problems and ways of reformation "Economic Science of modern Russia", № 3, in Russian
34. Shevyakov A., Kiruta A. (2002). *Measuring economic inequality* Moscow, "Leto", in Russian
35. Stiglitz J.E.(2002). *Globalization and Its Discontents*, W.W. Norton & Company, New York, London.
36. Sundstrom W. *Addressing Disparities in Income and Wealth* <http://www.scu.edu/ethics/publications/ethicsoutlook/2003/income>
37. *The social situation and living standards of Russia's population* (2004). Moscow, Goskomstat, in Russian.
38. Varshavsky A.E. (2003). The economic inequality is the determinative of slowing transition to the knowledge society. *Proceedings of the conference "Mathematics, computers, education"*, Moscow-Ijevsk, v.10, in Russian
39. Varshavsky A.E. (2007a). Slowing down diffusion of innovations and transition to knowledge

- society under in-come inequality growth. Konceptii, No 2, in Russian
40. Varshavsky A.E. (2007b). "Significant Reducing Income Inequality – a Critical Condition for Transition to the Innovative, Knowledge-based Economy". Economics and Mathematical Methods (Economika i Matematicheskie Metody), №4, in Russian
  41. Varshavsky A.E. (2007c). Meritocracy and development of the knowledge society: models and methods of investigation in "System modeling of social-economic processes" The Materials of the 30 th international scientific school-seminar / Eds. V. Grebennikov, I. Shchepina, V.Eitingon. Voronezh, Publishing and printing center of Voronezh State University, in Russian.
  42. Varshavsky A.E. and Kosmacheva O. (2003). Low-dimension aggregate macro-model of the Russian economy on the basis of annual data. Moscow, CEMI, Russian academy of sciences, WP/2003/159, in Russian
  43. Varshavsky Alexander. Basic income and increasing income inequality in Russia. 12th BIEN Congress 2008 - Dublin, Ireland. Inequality and Development in a Globalised Economy - The Basic Income Option . <http://www.basicincomeireland.com>
  44. Varshavsky A.E. Technological progress in the models of economic development. Moscow, Finansy&Statistika, 1984.
  45. Varshavsky A.E. Questionable Innovations: Risks and Responsibility / Working paper # WP/2009/255. Moscow, CEMI Russian Academy of Sciences, 2009.-113 p. (Rus.)
  46. VCIOM (2008). <http://wciom.ru/> VCIOM, № 967, 30.05.2008, in Russian .
  47. Weinberg D. A Brief Look at Postwar U.S. Income Inequality. <http://www.census.gov/prod/1/pop/p60-191.pdf>
  48. World Development Indicators 2004 (2004). The World Bank.
  49. World Income Inequality Database (2006),

### NELYGYBĖS RACIONALIŲ RIBŲ VERTINIMAS TAIKANT NAUJŲ PAJAMŲ PASKIRSTYMO MODELĮ

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**Santrauka.** Pirmame straipsnio skirsnyje apžvelgiamas pajamų nelygybės reiškinys Rusijoje. Toliau autorius pateikia naują pajamų paskirstymo modelį ir nagrinėja jo pritaikymo galimybes sprendžiant nelygybės problemas. Metodas grindžiamas prielaida, kad galima sudaryti specifinę seką, paremtą polinomu, kurio visos šaknys būtų viename rate ir apibūdinamos tam tikru pajamų rodiklio parametru. Darbas yra rimtas įnašas į pagrindinę matavimų teoriją bei nelygybės analizę. Tai atskleidžiama naudojant specialiai sudarytą naudingumo funkciją bei taikant Shapley reikšmių apskaičiavimus ir dinaminis modelius, iš kurių plaukia, kad prioritetinis nelygybės lygis išvystytoms šalims būtų tarp 0,25 ir 0,40 Gini koeficiento. Gauti rezultatai rodo, kad etikos problemos šiuo metu yra svarbiausios tvariai ekonominei plėtrai užtikrinti.

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