ISSN 1822-8011 (print)
ISSN 1822-8038 (online)
INTELEKTINĖ EKONOMIKA
INTELLECTUAL ECONOMICS
2010, No. 2(8), p. 7–20

MATHEMATICAL MODELLING OF THE EFFECTS OF URBANIZATION AND POPULATION GROWTH ON AGRICULTURAL ECONOMICS

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Abstract. Population growth often requires urbanization for both cultural and economic developments. However, there is a downside of this colourful scenario. Growing population requires more food consumption and urbanization, which requires more building construction, more roads, more shopping centers, more hospitals, more recreational centers, etc. To fulfill these demands, often lands for agriculture and ponds for fisheries, which should enhance food supply to meet demands for foods of vital significance, are being used to help urbanization. This could inevitably cause a disaster to a nation. The burning question is: how population, urbanization and food production can a1l be balanced. A modest attempt has been undertaken in this work to look for a mathematical solution. The authors have developed a mathematical model, which validates the possibility of an acute shortage of food production if the growth of population and rampant urbanization are not kept under control. Certain parameters related to the growth of population, urbanization and their impact on food production are taken into consideration. The model consists of three ordinary differential equations dealing with the interactions between these three interactive economies. It has been assumed that while population grows independent of urbanization and agricultural products, the rate of urbanization depends on the rate of growth of population, and food productions depend on both urbanization and population. To avoid any catastrophic shortage of food, an extra term has been added to the rate of the growth of food production. This could well represent food substitutes, urban gardening and/or importing food from other areas. By solving this system mathematically, it has been found that unless that extra source of food supply is kept in place, there could be a catastrophic shortage of food if both population and urbanization are uncontrolled. This simplistic model has established a strong qualitative agreement with real world scenario. The challenge is now to find the statistical estimates of the parameters in the model to fit the different agricultural economy of different parts of the world and predict the optimization of urbanization accordingly. Furthermore, availability of food does not necessarily make it affordable. People should be able to afford the price of their necessary food. An attempt has been made to estimate this price in terms of available agricultural production and the population who depend on it.

JEL Clasification:
Keywords: mathematical modeling, parameters of the growth of population, food interactive economies production.
Reikšminiai žodžiai: matematinis modeliavimas, gyventojų augimo parametrai, interaktyvios maisto gamybos ekonomikos.

1. Introduction

Food is life. When the price of a commodity goes up, its demand goes down. This fundamental principle of Economics is not applicable to food, especially to staple food like baby-milk, rice, wheat etc. Demand stays relatively steady even when price goes up. In fact when price of a staple food goes up, demand goes up, because of "panic-buying." Hoarding increases causing demand to go up further. That causes scarcity of food followed by more panic-buying, more hoarding and more demand and higher price. Such scenarios are not uncommon in underdeveloped countries and were not uncommon in developing or even developed countries. They forced governments to take proper actions. In India "fair-price" retail stores for staple food grew up in every neighborhood. Staple food is still rationed to control hoarding and artificial price hike. Thus food should never be treated like other commodities for example, clothes, furniture, automobiles, etc.

The primary source of food is agriculture which marked the beginning of human civilization. It was possibly the greatest discovery done by cave-women who, while taking care of children when men were gone for hunting for days, were forced to live on fruits and discovered that from seeds similar plants sprung up. So agriculture is the root of human civilization. Thus any form of human activity which could hinder agriculture must be analyzed carefully. If it cannot be eliminated, it must be kept under a vigilant control. Urbanization is one of such activities. Societies often welcome growth without looking into all the pros and cons. This is the topic of our study. A mathematical approach has been taken under consideration.

Urbanization is essential to grow the economy. People of developed countries look for more economic developments, and rich people of developing and underdeveloped countries try to imitate them often without paying any attention to the national interests one of which is "feed the hungry." They try to urbanize agricultural lands and that affects agricultural output negatively often leading to critical food shortage.

According to reports of FAO (Food and Agricultural Organization), "World population is projected to rise to 9.1 billion in 2050 from a current 6.7 billion requiring a 70% increase in farm production." [1], also global hunger is still on the rise. More than one billion people are undernourished; about 1.02 billion people going hungry everyday [2]. According to World Health Report [3], "Hunger is no. 1 on the list of the world's top 10 health risks." Hunger kills more people every year than AIDS, malaria and tuberculosis combined [4].

According to Sir Michael Atiyah, President, of the Royal Society of London [5], "World population is growing at the unprecedented rate of almost 100 million people every year and human activities are producing major changes in the global environment," detrimental to agriculture. Economic prosperity being the most natural trend of human pursuit, in creasingly, people all over the world tend to urbanize rural lands. Many scientists consider food security as a major challenge closely connected to urbanization [6]. Even in the United States which is one of the biggest suppliers of food, USDA (U.S. Department of Agriculture) is closely monitoring loss of farmland due to urbanization. ERS (Economic Research Service of the USDA) is in touch with the concerned groups of citizens. USA is an urban population. Nevertheless, most people attempt to maintain all the primary rural characteristics. Family gardening is a hobby of the general mass. According to ERS [7], "Despite the efforts to preserve farmland, the continued encroachment of new urban areas into agricultural regions creates a tension between the new inhabitants and the agricultural production operations that struggle to survive in the same location." We have here Farmland Protection Policy Act 7, U.S.C. §§4201–4209 and Farmland Protection Program, 7 U.S.C. §§3838h-3838j. Several schemes of studies on various aspects of urbanization have also been developed and applied by different groups of engineers and geographers [8, 9, 10, 11, 12].

In Africa and in most Asian countries including India and China situations are quite different. A statistical report [13], shows that more than half of the world's 6.7 billion people live in urban areas which covers only 3% of land of the earth. But to have all the available modern amenities, they use the most energy, cause the most pollution, and convert most farmland for factories and fashion houses. "Much cultivated land in China was lost due to construction of dams for hydropower generation." [14]. Although, compared to America, China's urbanization rate is much lower and slower, yet it is having a negative impact on the rural economy of China [15]. "Loss of agricultural land to human settlements is far more serious in India." [16]. From 1955 through 1985 about 1.5 million hectares of land, mostly agricultural, went to urban growth and over a million hectares more have been converted into urban areas. So urbanization is affecting the amount of crops that India must produce to feed her growing population. In [17], using 49 years of data, it has been found that in Andhra Pradesh, one of the bread baskets of India, urbanization is causing a severe food security problem.

Almost all studies have been conducted by analyzing statistical data. In this article, mathematical modeling has been introduced to study this issue from a broader perspective. We have considered the logistic equation for population growth, assumed a linear relationship between urbanization and population and considered how growth of agriculture is affected positively by efforts of farmers and negatively by natural forces and urbanization. To understand the scenario of demand and supply and pricing of agricultural produce, we have included in the equation for growth of produce a term which could increase the amount of produce when it is positive and decrease the amount of produce when it is negative. Although this theory is general, we are primarily interested to study availability and affordability of staple food like rice, wheat, etc. This is what we call the agricultural economy. Urbaniza- tion affects both availability and affordability of food. With rapid industrialization as urban economy gets better, more urbanization follows, more farmland is lost, and rural economy suffers. More people migrate to urban areas, and if the nation's economy cannot afford that, rural people migrating to urban areas find no jobs, no shelter, and no food. This is already happening in many developing and underdeveloped countries. This is not only detrimental to the world from a humane point of view, but also from the standpoint of global economy. Loss of rural economy simply means loss of agricultural products.

Food is absolutely essential to sustain life. We must eat food in order to live. Even if food price increases, demand does not decrease On the contrary, often being scared of the fact that price could go up further, people start "panic buying" especially rice and wheat, which could be stored for a few months. This creates a food shortage. Food movement starts and people get unruly. This happened in West Bengal, India in 1959. Chaos in economy bleeds into chaos in politics, which often results in economic crisis globally.

With a mathematical model food crisis could be predicted with a certain amount of accuracy. Statistical data should enhance the robustness and reliability of such models. All parameters of our model should be evaluated by statistics. In fact, such economic modeling requires group involvements. It should be noted that while statistical analysis and predictions are somewhat local [17], the mathematical model is global requiring initial data from the country where it will be applied.

In our model, there are three ordinary differential equations. Two are nonlinear. The third equation for the growth of produce, coupled with urbanization and population fol- lows the pattern of a chemical reaction-type equation [18, 27]. These equations were solved analytically and results have strong qualitative agreement with the statistical investigation presented in [17].

2. The Model

Let $p(t)$, $u(t)$, and $f(t)$ be three dependent variables representing population, urbanization and food production, all depending on time t. It has been assumed that the growth of population is practically independent of $u(t)$ and $f(t)$. This may however seem to be too simplistic. But for this work this assumption, based upon observations on developing and some underdeveloped countries, has been maintained in this article.

First Euler, and later Malthus suggested a model for population growth as

$$\frac{dp}{dt} = \alpha p, \quad \alpha > 0 \tag{1}$$

giving

$$p(t) = p_0 e^{\alpha t},\ p_0 = p(0) \tag{2}$$

This is too simplistic and as Murray [24] said "pretty unrealistic."

Later Verhulst [25] suggested the well known logistic model

$$\frac{dp}{dt} = \alpha p(1 - p/K) \tag{3}$$

at $t = 0$ $p = p_0$.

Solving we get

$$p(t) = \frac{Me^{\alpha t}}{1 + (M/K)e^{\alpha t}} \tag{4}$$

where

$$M = \frac{p_0}{1 - p_0/K} \tag{5}$$

$$\lim_{t \to \infty} p(t) = K. \tag{6}$$

The logistic equation (3) is still being extensively used with some modification in the population theory for various species [24].

The law of entropy in Informatics suggests that as population grows a census of a nation will miss some number of people which is likely to be caused mostly by the dynamics of the very system of recording the census. So, let us assume

$$p(t) = P(t) + \varepsilon(t) \tag{7}$$

where $P(t)$ is recorded by the census and $\varepsilon(t)$ is the error in the record. Since census gets better as time goes on, because of the efforts given by the statisticians and government mandates, we may assume that $\varepsilon(t)$ satisfies the following properties: (i) $\varepsilon(t) > 0\ \forall t$, (ii) $\varepsilon'(t) < 0$, which implies that $\varepsilon(t)$ is a decreasing function of t and (iii) $\lim_{t\to\infty}\varepsilon(t)=0$.

Then, let us assume that $P(t)$ satisfies the logistic equation,

$$\frac{dP}{dt}=\alpha P(1-P/K),\quad \alpha>0 \tag{8}$$

and

$$\frac{d\varepsilon}{dt}=-m\varepsilon,\quad m>0 \tag{9}$$

At $t = 0$, $P = P_0$ and $\varepsilon = \varepsilon_0$ from (7)

$$p_0 = P_0 + \varepsilon_0 \tag{10}$$

and

$$\frac{dp}{dt}=\alpha P(1-P/P)-m\varepsilon \tag{11}$$

A portion of population lives in the urban areas. Then the rate of change of urbanization is proportional to the rate of change of population.

Then,

$$\frac{du}{dt}=\lambda\frac{dp}{dt} \tag{12}$$

At $t=0$, $u=u_0=\lambda p_0$.

From (11)

$$\left.\frac{dp}{dt}\right|_{t=0}=\alpha P_0\left(1-\frac{P_0}{K}\right)-m\varepsilon_0. \tag{13}$$

In order that $p(t)$ is an increasing function at $t = 0$,

$$\alpha P_0(1 - P_0/K) > m\,\varepsilon_0 \tag{14}$$

Then,

$$\alpha > m\,\varepsilon_0/P_0\,(1 - P_0/K) \tag{15}$$

If we assume K, which is given by (from (6)) as the limit of $p(t)$ as well as $P(t)$

$$\lim_{t\to\infty} p(t)=K \tag{16}$$

Let $\varepsilon_0 = qP_0$ and $K = rP_0$, $r > 1$. Then from (15)

$$\alpha\frac{m\cdot q}{1-1/r} \tag{17}$$

If $m = 0.01$, $q = 0.01$, $r = 10$

$$\alpha > 0.0111 \tag{18}$$

so that population may increase at $t = 0$. If population is small, like some of the northern European countries, both m and q will be small. Then α could be much less than 1%, which means rate of growth of population could be lot less than 1%. Hence we will assume that at $t = 0$, $p'(t)$ is positive and $p(t)$ is increasing.

Now, rate of change of $f = X - Y - Z + W$.

Where, X = increase of f per unit of time,
Y = loss of f due to pests, insects, and natural disasters per unit time.
Z = loss of f due to urbanization per unit time.
W = increase of f due to urban gardening, national/international relief as (food substitute) needed because of population growth per unit time.

Thus,

$$\frac{df}{dt}=\gamma f-\delta f-\phi f\cdot u+\theta f\cdot p \tag{19}$$

where, $X = \gamma f$, $Y = \delta f$, $Z = \phi f u$, $W = \theta f p$

γ = % of growth of f per unit time;
δ = % of loss of f per unit time;
ϕ = % of loss of f per unit of u per unit time;
θ = % of growth of f supplied per unit of p per unit time.

At $t = 0$, $f = f_0$, solution of (19) gives the amount of food available for human consumption.

Since $u = \lambda p$

$$\frac{df}{dt}=(\gamma-\delta+\eta p)f \tag{20}$$

where

$$\eta=\theta-\lambda\phi. \tag{21}$$

At $t = 0$,

$$\left.\frac{df}{dt}\right|_{t=0}=(\gamma-\delta+\eta p_0)\cdot f_0. \tag{22}$$

Since $f_0 > 0$, $p_0 > 0$, if $\gamma > \delta$ and $\eta > 0$.

$$\left.\frac{df}{dt}\right|_{t=0}>0 \tag{23}$$

making f and increasing function at $t = 0$.

If $\gamma < \delta - \eta p_0$, $\frac{df}{dt}$ at $t = 0$ will be a decreasing function, and food crisis will start from the very beginning. This also could happen if $\eta < 0$, which implies $\theta < \lambda\phi$, or in otherwords,

the amount of food produced by urban gardening and imports is less than loss of food due to urbanization. An interesting observation is if θ is negative.

$$\frac{df}{dt} = (\gamma - \delta - (\theta + \lambda\phi)p) f. \quad (24)$$

If p is kept fixed (only theoretically) then f will grow iff

$$\gamma > \delta + (\theta + \lambda\phi) p \quad (25)$$

This is practically impossible, because very high yields of crops will be needed, requiring vast resources of agricultural lands, equipment and manpower! Thus starvation is inevitable. This is the economic reality of famine.

Thus the simple model that we have consisting of equations (11), (12), and (19) subject to the initial conditions at $t = 0$, $p = p_0$, $u = u_0$ and $f = f_0$, has already resulted in some practical conclusions. Now we must solve them for more answers with economic significance.

3. Analytical Solution

From (7) we can construct

$$\frac{dp}{dt} = \frac{dP}{dt} + \frac{d\varepsilon}{dt} \quad (26)$$

From (4) and (26)

$$P = \frac{Me^{\alpha t}}{1 + (M/K)e^{\alpha t}} \quad (27)$$

where

$$M = P_0/(1 - P_0/K) \quad (28)$$

If $\beta = M/K$, then

$$\beta = (P_0/K)/(1 - P_0/K) \quad (29)$$

This gives

$$P_0/K = \beta/(1+\beta) \quad (30)$$

From (9)

$$\varepsilon(t) = \varepsilon_0 e^{-mt} \quad (31)$$

m is small for a slow exponential decay. Thus

$$p(t) = P(t) + \varepsilon(t) \quad (32)$$

$$= \frac{K\beta e^{\alpha t}}{1 + \beta e^{\alpha t}} + \varepsilon_0 e^{-mt}$$

It must be noted that α and m are both rate constants, one indicates increase of population while the other indicates the decrease of error in the census.

It is possible to choose other forms of $\varepsilon(t)$. But all must satisfy properties of $\varepsilon(t)$ discussed before. In these days of super advancement of technology, it is quite likely that the error in the census must decrease exponentially.

Urbanization depends only on population and as such

$$u(t) = \lambda p(t) \quad (33)$$

From (20)

$$\int_{f_0}^{f} \frac{df}{f} = \int_0^t (\gamma - \delta) dt + \eta \int_0^t p dt \quad (34)$$

$$\ln(f/f_0) = (\gamma - \delta)t + \eta(I_1 + I_2)$$

where

$$I_1 = K \int_0^t \beta e^{\alpha t}/(1 + \beta e^{\alpha t}) \cdot dt$$

$$= (K/\alpha) \cdot \ln\left\{\frac{1 + \beta e^{\alpha t}}{1 + \beta}\right\}$$

and

$$I_2 = \varepsilon_0 \cdot \int_0^t e^{mt} dt$$

$$= (\varepsilon_0/m)(1 - E^{-mt})$$

Then from (34)

$$f/f_0 = e^{G(t)} \quad (35)$$

where

$$G(t) = (\gamma - \delta)t + (K\eta/\alpha)\ln\left\{\frac{1 + \beta e^{\alpha t}}{1 + \beta}\right\} + a \cdot \eta(1 - e^{-mt}) \quad (36)$$

where

$$a = \varepsilon_0 / m.$$

4. Analysis of the Model

From (36), $G(0) = 0$, which satisfies the condition that

$$f(0) = f_0 e^{G(0)} = f_0. \quad (37)$$

From (36), $f = e^{G(t)}$, gives the amount of food available for consumption at a time t.

From (36),

$$f'(t)=f_0e^{G(t)}\cdot G'(t) \tag{38}$$

$$f''(t)=f_0e^{G(t)}\left\{G'(t)^2+G''(t)\right\} \tag{39}$$

Also,

$$G'(t)=(\gamma-\delta)+(K\eta\beta e^{\alpha t})/(1+\beta e^{\alpha t}) +\varepsilon_0\eta e^{-mt} \tag{40}$$

$$G'(0)=(\gamma-\delta)+K\eta\beta/(1+\beta)+\varepsilon_0\beta$$
$$=\gamma-\delta+\eta p_0 \quad \text{from (32)}$$

In general, from (32) and (40),

$$G'(t)=\gamma-\delta+\eta p(t) \tag{41}$$

Also,

$$G''(t)=\eta\left\{K\alpha\cdot\frac{\beta e^{\alpha t}}{(1+\beta e^{\alpha t})^2}-m\varepsilon_0e^{-mt}\right\} \tag{42}$$
$$=\eta p'(t)$$

Since we assume that population is growing, $p'(t) > 0$.

Case 1. If $\theta > \lambda\phi$, $\eta > 0$, then since $p(t) > 0$, $G'(t) > 0$ if $\gamma > \delta$, which is true if the rate constant for food production, exceeds the rate constant for loss of crops due to natural disasters and/or pest infiltrations. $G(t)$ is then an increasing function.

$\theta > \lambda\phi$ implies that amount of food imported and/or produced by urban gardening exceeds the amount of loss of food due to urbanization. Larger values of $p(t)$ will increase $G'(t)$ which means $G(t)$ will increase faster. However, it will also increase the ratio p/p_0 which will affect (38) adversely.

Also if δ is large such that $\gamma + \eta p_0 = \delta$, from (43) $G'(0) = 0$ then, since $p'(0) > 0$ ($p(t)$ increasing at $t = 0$), from (45) $G''(0) > 0$. This means $G(0)$ is a minimum; and $G(t)$ is increasing if $t > 0$.

Case 2. $\theta = \lambda\phi$, or $\eta = 0$. Then $G'(t) = \gamma - \delta$. Then $G(t)$ is an increasing function. Then population increase has no effect on food production. Thus, in this case our model fails. However, our model is based upon data collected over a period of time and during that period it is very unlikely that θ will remain equal to $\lambda\phi$.

Case 3. $\theta < \lambda\phi$, or $\eta < 0$, then in order that $G(t)$ is an increasing function, $\gamma > \delta + \omega p(t)$ where $\omega = -\eta > 0$. If this condition fails, food crisis is inevitable. Also, in case $G'(t) = 0$ at some t, at that point $G(t)$ will have a maximum because η being negative, from (45), for all t, $G''(t) < 0$. Thus when $\eta < 0$, $G(t)$ will decrease causing scarcity of food as time progresses.

5. Non-Dimensional Form

First we need to nondimensionalize the equations, so that we may use all parameters as scalars and the scalar results will represent the results to be given in any appropriate units that a user may use.

Let T = time, M = population (mass of people), F = unit of food.

Then,

t' = nondimensional time $=t/T$;

p' = nondimensional population $= p/M$, f' = nondimensional quantity of food $= f/F$;

$$K' = K/M,\quad \alpha'=\alpha/T^{-1},\quad \gamma'=\gamma/T^{-1},\quad \delta'=\delta/T^{-1}$$
$$\eta' = \eta/(M^{-1}T^{-1}),\quad \varepsilon'=\varepsilon/M \tag{43}$$
$$m' = m/T^{-1}, \beta \text{ is a scalar}.$$

Then $\alpha't'=\alpha t, m't'=mt$; and from (32)

$$p'(t)M=\frac{K'M\beta\cdot e^{\alpha't'}}{1+\beta e^{\alpha't'}}+\varepsilon_0'M\cdot e^{-m't'}$$

giving

$$p'(t)=\frac{K'\beta e^{\alpha't'}}{1+\beta e^{\alpha't'}}+\varepsilon_0'e^{-m't'} \tag{44}$$

Since (32) and (44) are alike, we may plot/compute (32) considering it as a scalar equation independent of the units of time and population to be used.

In (36), f/f_0 is scalar. So, $G(t)$ must be a scalar. Let us consider each term in $G(t)$. Nothing that β is a scalar and $\alpha t = \alpha' t'$, and $mt = m' t'$,

$$(\gamma-\delta)t=(\gamma'\cdot T^{-1}-\delta'\cdot T^{-1})\cdot t'\cdot T$$
$$=(\gamma'-\delta')t'.$$
$$K\eta/\alpha=(K'\cdot M\cdot\eta'\cdot M^{-1}T^{-1})/(\alpha'\cdot T^{-1})$$
$$=K'\eta'/\alpha'$$
$$a\eta\quad=(\varepsilon_0/m)=(\varepsilon_0'\cdot M)/(m'\cdot T^{-1})\cdot(\eta'\cdot M^{-1}\cdot T^{-1})$$
$$=(\varepsilon_0'/m')\eta'=a'\cdot\eta'.$$

Thus $G(t)$ given by (37) may be considered as a scalar function. Thus we may compute $G(t)$ in any unit of time.

Finally, let us nondimensionalize (20). From (43) $f = f'\cdot F$, $t = t'\cdot T$, $\gamma = \gamma'\cdot T^1$, $\delta = \delta'\cdot T^1$, $\eta = \eta'\cdot M^1\cdot\cdot T^1$, $p = p'\cdot M$, giving $(F\cdot T^1)\frac{df'}{dt} = (\gamma'\cdot T^1 - \delta'\cdot T^1 + \eta'\cdot M^1\cdot T^1\cdot p'\cdot M)\cdot(f'\cdot F)$ which is reduced to

$$\frac{df'}{dt} = (\gamma' - \delta' + \eta' \cdot p') \cdot f' \quad (45)$$

This equation and (20) are alike. Therefore we may solve (20) considering all the variables as scalars. And the same is true for (11). The solutions are $f(t)$ and $p(t)$ which are scalars.

Let

$$N = f/p \quad (46)$$

N is called the Economic Indicator of Food Availability (EIFA).

Obviously, if $N > 1$, more food is available than the number of population, and if $N < 1$, there is less amount of food and more number of people. However, a question may arise whether one unit of f is sufficient to feed one unit of p. So let us define one unit of f to be that much amount of food which is sufficient for one unit of p. This unit may vary from state to state and from country to country. This cannot affect our model which has been nondimensionalized. At the beginning, when $t = 0$, social workers should determine the value of N_0 given by f_0/p_0. If they find f_0 enough to feed p_0, they may set $N_0 = 1$; if f_0 can feed only 75% of p_0, then $N_0 = 0.75$, etc. This is an educated guess used all over the world, and social workers have been trained to figure out the value of f_0. To determine how acute the state of starvation is, and/or how much food is available for distribution, we need to compute N when N_0 is given.

From definition,

$$N / N_0 = \frac{f / p}{f_0 / p_0} = \frac{(f / f_0)}{(p / p_0)}$$

thus

$$N = \frac{(f / f_0)}{(p / p_0)} \cdot N_0. \quad (47)$$

From (35), we can compute f/f_0 and from (44)

$$p / p_0 = K\beta e^{\alpha t} / (1 + \beta e^{\alpha t}) + (\varepsilon_0 / p_0) \cdot e^{-mt} \quad (48)$$

Thus N is computed when N_0 is known. Several sets of data for α, λ, γ, δ, ϕ and θ have been used and results on a five year basis are recorded in table 1 and 2.

6. Estimation of Price

According to the standard macro as well as micro economic theory on demand and supply, if D = demand, S = supply, and r = price, then,

$$\text{Price Elasticity of Demand} = \frac{dD}{dr} < 0$$

and,

$$\text{Price Elasticity of Supply} = \frac{dS}{dr} > 0.$$

From keen observations and analysis of events related to demand and supply of basic, staple foods like baby-formula, rice and wheat, we have found that those basic laws of demand and supply of economics are not applicable to food. Food is no ordinary commodity, it is life.

Food prices hardly drop. In general, it increases slowly during the regular time, and fast when food is scarce. One major factor for that is the growth of population. Demand of food is always on the rise. As price increases, demand still increases. On the supply side, there are limitations too. Suppliers are limited by the amount of production of food which is primarily determined by the agricultural output. Thus $\left|\frac{dD}{dr}\right|$ and $\left|\frac{dS}{dr}\right|$ do not change very much. Demand and supply are quite inelastic.

In most developing countries, including India and China and in almost all underdeveloped countries, many people cannot afford to buy refrigerators and freezers to store food. Thus they cannot store much food except grains. Food often gets spoiled. That also drives up food price. Another economic reality is that availability of a commodity does not necessaril mean affordability of it. The price must be affordable. Thus we will now make an effort to estimate food price in terms of N, the Economic Indicator of Food Availability.

Sample equations for demand and supply theory determine price of a commodity. This theory may be expressed as follows:

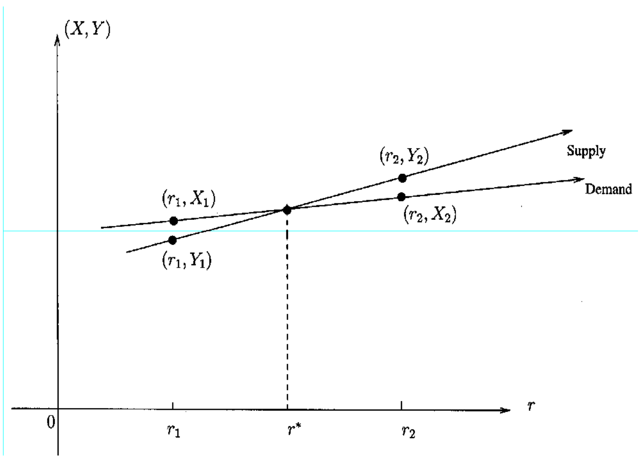

Figure 1. Sample equations for demand and supply theory

Let the following data be given:

Price	Demand	Supply
r_1	X_1	Y_1
r_2	X_2	Y_2

Then the equations for the lines of demand and supply will be:

$$X = X_1 + \frac{X_1 - X_2}{r_1 - r_2}(r - r_1) \tag{49}$$

$$Y = Y_1 + \frac{Y_1 - Y_2}{r_1 - r_2}(r - r_1) \tag{50}$$

respectively, where $Y_1 < X_1$ and $Y_2 > X_2$, $r_2 > r_1$

Price is determined where demand and supply have intersected. If r^* is the price

$$r^* = r_1 + \frac{X_1 - Y_1}{(X_1 - Y_1) + (Y_2 - X_2)} \cdot (r_2 - r_2) \tag{51}$$

If, $X_1 = Y_1$, $r^* = r_1$ and if $X_2 = Y_2$, $r^* = r_2$. From (51) $r_1 < r^* < r_2$.

To make an estimation of r^* in terms of N, let us estimate demand and supply by proposing that one unit of demand of an agricultural produce is for one unit of population (like 10 leaves of bread are needed by 10 people) and one unit of supply is equivalent to one unit of agricultural produce for one unit of population (like 8 loaves of bread supplied to 8 persons). Thus demand will be approximated by population and supply approximated by the amount of produce. This assumption is quite logical with regard to food, especially staple food like rice and wheat, sold at "Fair-price" retail stores. However, this assumption does not seem to follow the standard norms of micro and macroeconomics. Of course, the economic conditions of individuals and the economic infrastructure of a country based upon GDP (Gross Domestic Products) do have some impact on food price, yet regardless of how we see it, "basic food" and "hunger" go hand-in-hand and depend on population and supply of food.

It is undeniable that there are many factors that affect the food-price [30]. But we would like to take a simplistic approach with regard to our model. At each time step $t = t_n (n = 1, 2, ...)$, we compute $f(t_n)/f_0$ and $p(t_n)/p_0$. Also

$$N(t_n) = \frac{f(t_n)}{p(t_n)}$$

Obviously, $N(t_n)$ gives $f(t_n)$ as a percentage of $p(t_n)$.

At any given time frame $t = t_n, p(t_n)$ is population at $t = t_n$. Thus $p(t_n)$ is a constant at $t = t_n$. Thus even though the price of food may vary at $t = t_n$, depending upon the amount of supply, the number of consumers remain the same. Thus we may assume that demand stays constant while supply could vary. According to our model, availability becomes less when

$\theta = 0$ (no food substitute and/or no food import) and when $\theta > 0$ (food substitute and/or food import is implemented).

Let supply$\Big|_{\theta=0, t=t_n} = f_1(t_n) = f_1^n$

and supply$\Big|_{\theta>0, t=t_n} = f_2(t_n) = f_2^n$

and Demand$\Big|_{t=t_n} = p(t_n) = p^n$. The equation of demand is then,

$$p = p^n \tag{52}$$

and the equation of supply is:

$$r^n = r_2^n + \frac{(f^n - f_2^n)}{f_2^n - f_1^n}(r_2^n - r_1^n) \tag{53}$$

where for the price of r_i^n, the supply is $f_i^n (i = 1,2)$ and demand is p^n for both $r_i^n (i = 1,2)$.

If the price of the commodity is r^n*, then at that price $f^n = p^n$. Giving,

$$r^{n*} = r_2^n + \omega(r_2^n - r_1^n) \tag{54}$$

where

$$\omega = \frac{(p^n - f_2^n)}{f_2^n - f_1^n} \tag{55}$$

Now, by definition, for $\theta = 0$, if the value of N is N_1, then at $t = t_n$,

$$N_1^n = f_1^n / p^n$$

Then from (55)

$$\omega = (1 - N_2^n)/(N_2^n - N_1^n) \tag{56}$$

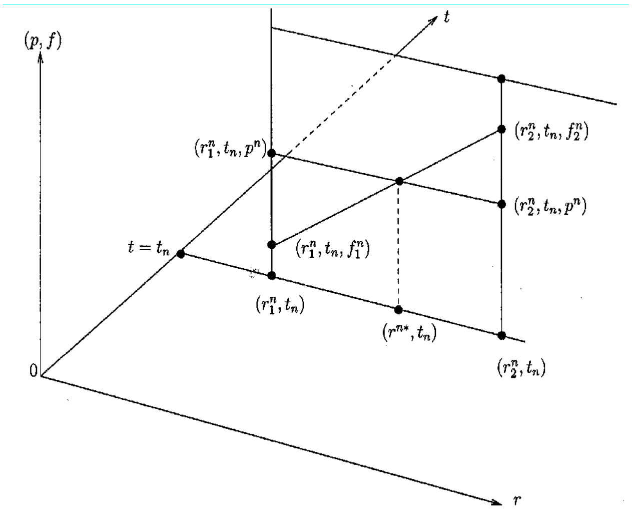

Figure 2.

By definition, for all i, $N_i^n > 0$. At $r^{n}*$, demand = supply, giving $N^{n}* = 1$. Since $f_2^n > f_1^n$ and p^n is a constant

$$N_2^n > N_1^n \tag{57}$$

If $N_2^n = 1$ which implies $N_1^n < 1$, $\omega = 0, r^{n*} = r_2^n$.

If $N_1^n = 1$ mplying $N_2^n > 1$, $\omega = -1$, $r^{n*} = r_1^n$.

If $N_1^n < 1 < N_2^n, \omega < 0$

$$\omega = -(N_2^n - 1)/(N_2^n - N_1^n)$$

$$r^{n*} = \frac{1 - N_1^n}{N_2^n - N_1^n} r_2^n + \frac{N_2^n - 1}{N_2^n - N_1^n} r_1^n \tag{58}$$

Now,

$$\frac{1 - N_1^n}{N_2^n - N_1^n} r_2^n + \frac{N_2^n - 1}{N_2^n - N_1^n} r_1^n > r_1^n$$

is true if,

$$\frac{1 - N_1^n}{N_2^n - N_1^n} r_2^n > \left(1 - \frac{N_2^n - 1}{N_2^n - N_1^n}\right) r_1^n$$

or,

$$\left(\frac{1 - N_1^n}{N_2^n - N_1^n}\right) r_2^n > \left(\frac{1 - N_1^n}{N_2^n - N_1^n}\right) r_1^n$$

or

$$r_2^n > r_1^n$$

is true.

Thus $r^{n*} > r_1^n$. Similarly we can prove that $r^{n*} < r_2^n$. Hence

$$r_1^n < r^{n*} < r_2^n \tag{59}$$

This inequality is almost obvious from the Figure 2.

If $N_1^n > 1$, which implies $N_2^n > N_1^n > 1$, then $\omega < 0$.

$$r^{n*} = -\frac{N_1^n - 1}{N_2^n - N_1^n} r_2^n + \frac{N_2^n - 1}{N_2^n - N_1^n} r_1^n \tag{60}$$

Now,

$$-\frac{N_1^n - 1}{N_2^n - N_1^n} r_2^n + \frac{N_2^n - 1}{N_2^n - N_1^n} r_1^n < r_1^n$$

is true if

$$\left(\frac{N_1^n - 1}{N_2^n - N_1^n} - 1\right) r_1^n < \frac{N_1^n - 1}{N_2^n - N_1^n} r_2^n$$

or,

$$\frac{N_1^n - 1}{N_2^n - N_1^n} \cdot r_1^n < \frac{N_1^n - 1}{N_2^n - N_1^n} \cdot r_2^n$$

or

$$r_2^n < r_1^n$$

is true. But this is true. Hence in this case

$$r^{n*} < r_1^n < r_2^n \tag{61}$$

This is also obvious from the fact that at $t = t_n$ if $N_1^n > 1$, that means at the price level $r_1^n, f_1^n > p^n$, which implies supply exceeds demand, therefore price must be lowered or in otherwords $r^{n*} < r_1^n$. This may not be done. Under such conditions, suppliers try to store more food for bad days or export food to other places where that food is in demand.

The worst scenario is if

$$N_2^n < 1.$$

Then $N_1^n < N_2^n < 1$, and $\omega > 0$.

$$r^{*,n} = \frac{1 - N_1^n}{N_2^n - N_1^n} r_2^n - \frac{1 - N_1^n}{N_2^n - N_1^n} \cdot r_1^n$$

$$> r_2^n.$$

Thus,

$$r^{*,n} > r_2^n > r_1^n. \tag{62}$$

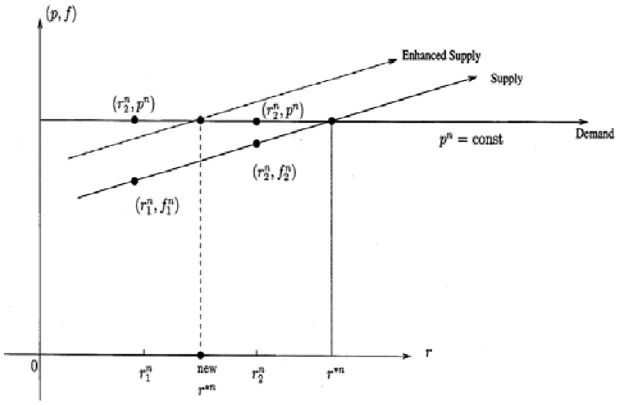

Figure 3.

This causes economic hardship measured by the amount which is at least $(r^{*n} - r_2^n)$. To reduce this amount food imports/substitutes are needed.

All these discussions are related to the macro economics with a broad general view about demand and supply of food. On the basis of the micro economics, related to individual consumers, demand may fluctuate because at any given t_n population could fluctuate. Inorder to study the state of economy of agriculture, we will adopt the simple perturbation principle.

Let at $r = r_1^n, p^n = p_1^n$ and at

$r = r_2^n, p^n = p_2^n = p_1^n + \varepsilon$. Where ε is small and could be positive or negative. Then at equilibrium, where demand equals supply, price is given by:

$$r^n = r_2^n + \frac{p_2^n - f_2^n}{(f_2^n - f_1^n) - (p_2^n - p_1^n)}(r_2^n - r_1^n) \quad (63)$$

$$= r_2^n + \frac{1 - (N_2^n - \varepsilon / p_1^n)}{(N_2^n - \varepsilon / p_1^n) - N_1^n}(r_2^n - r_1^n) \quad (64)$$

Obviously, $(N_2^n - \varepsilon / p_1^n)$ is approximately the same as N_2^n because p_1^n is significantly larger than ε. Hence,

$$r^n = r_2^n + \frac{1 - N_2^n}{N_2^n - N_1^n}(r_2^n - r_1^n) \quad (65)$$

This equation is the same as (54) where ω is given by (56). Thus when it comes to distribution and pricing of basic foods, macro economics and micro economics both show the same results. This makes pricing at Fair-Price Retail Stores both robust and relevant.

7. Applications of the parameters of *α, λ, γ, δ, ϕ* and *θ*

The parameter α = rate of growth population. Thus $\alpha = 0.05$ means 5% growth of population per unit time (which is one year, in general). λ = ratio of urban population and total population. $\lambda = 0.3$ means 30% of total population live in urban areas. γ the rate of growth of agriculture (food) irrespective of any external help or hurdle for agriculture. Thus $\gamma = 0.3$ means 30% growth of food per unit time. $\delta = 0.1$ implies 10% of food is destroyed due to natural and/or man-made disasters per unit of time. φ gives the percent of loss of agricultural produce per unit time per unit population due to urbanization. θ gives the percent of external food supply per unit time per unit population. It is assumed that this amount of food is for sale and not for free distribution. Also, $N = f/p$ means if $N = 0.8$, $f = 0.8 \cdot p$ means food is available for 80% of the people. For example on the average 100 families (each having 5 members) of Bengal consumes about 2500 kilograms of rice each year. Then food for 80% families means only 2000 kilograms are available at a fair price store. In many developing and underdeveloped countries there are several fair price stores for staple food like rice and wheat regulated and controlled by the government.

All of our computations are based upon the above assumptions.

8. Discussion of Results

In Table 1 and Table 2 several computational findings have been recorded. Values of N/N_0 determine both availability and affordability of food.

From the data sets #1, #2, and #3 of the Table 1, it may be noticed that for a 20% less urbanization (20% less urban population) value of N/N_0 is increased from 1.23 to 1.73, a 41% increase and if the rate of growth of population goes down from 10% to 5% N/N_0 increases by 44%

From the data set #3 and # 4, we notice that if 10% more population move to urban areas, N/N_0, over 5 years, is decreased from 2.15 to 1.99 a 7% decrease with a 5% rate of growth of population.

For all the above cases 1% food has been imported and/or substituted.

From the set #4 and #5 we notice that if 5% less people move to urban areas, and if θ is increased to 10% import, N/N_0 increases from 1.99 to 3.22 about 62% higher. We ought to remember that our Economic Indicator of Food Availability (EIFA) is N. So depending upon N_0, the true economic indicator N will be found.

Set #6 of Table 1, reveals a grim scene. With a 15% rate of growth of population, 35% urbanization, 15% rate of growth of agriculture and no food import there may be a massive starvation at the end of 5 years when N/N_0 = 0.68. To save people from starvation set #7 and set #8 have been recorded. These are self-explained with the discussions stated above.

In the Table 2, only effects of urbanization on N/N_0 have been recorded. With α = 0.15, γ = 0.2, δ = 0.075 and θ = 0 (no import). When urbanization has increased from 0% to 35%, N/N_0 dropped from N/N_0 from 0.99 to 0.68 over 5 years, which is a little over 31%.

All of our predictions are based upon estimated data collected from the references. Thus accuracy of all predictions are based upon accuracy of estimations. We welcome agricultural economists to provide us with better estimations.

One apparent drawback of this model is the estimation of K given by

$$\lim_{t\to\infty} p(t) = K = \lim_{t\to\infty} P(t).$$

For increasing values of K, $p(t)$ tends more towards an exponential growth due to Malthus in 1798. (This work was done earlier by Euler.) From (30) as K increases, β decreases, which affects $G(t)$ in (36) and hence $f(t)$.

To look into this we ran our code for α = 0.15, λ = 0.2, γ = 0.3, δ = 0.075, ϕ = 0.05 and θ = 0.1 for K = 10 and K = 30. After 5 years, f/f_0 increased from 5.80 to 5.94 (about 2%, p/p_0 increased from 1.89 to 2.0 (about 6%) and N/N_0 decreased from 3.06 to 2.92 (about 5%). For overpopulated countries these could be significant.

Table 1

t	f/f_0	p/p_0	N/N_0
Set # 1 $\alpha = 0.1$ $\lambda = 0.45$ $\gamma = 0.3$ $\delta = 0.1$ $\phi = 0.15$ $\theta = 0.01$			
1	1.1501	1.0928	1.0525
2	1.3155	1.1931	1.1026
3	1.4956	1.3013	1.1493
4	1.6894	1.4179	1.1915
5	1.8951	1.5432	1.2280
Set # 2 $\alpha = 0.1$ $\lambda = 0.25$ $\gamma = 0.3$ $\delta = 0.1$ $\phi = 0.05$ $\theta = 0.01$			
1	1.2182	1.0928	1.1148
2	1.4837	1.1931	1.2436
3	1.8065	1.3013	1.3882
4	2.1990	1.4179	1.5509
5	2.6760	1.5432	1.7340
Set # 3 $\alpha = 0.05$ $\lambda = 0.25$ $\gamma = 0.3$ $\delta = 0.1$ $\phi = 0.05$ $\theta = 0.01$			
1	1.2183	1.0454	1.1654
2	1.4841	1.0927	1.3582
3	1.8076	1.1418	1.5831
4	2.2013	1.1929	1.8454
5	2.6805	1.2459	2.1514
Set # 4 $\alpha = 0.05$ $\lambda = 0.35$ $\gamma = 0.3$ $\delta = 0.1$ $\phi = 0.075$ $\theta = 0.01$			
1	1.2013	1.0454	1.1491
2	1.4420	1.0927	1.3197
3	1.7296	1.1418	1.5148
4	2.0728	1.1929	1.7377
5	2.4821	1.2459	1.9921
Set # 5 $\alpha = 0.05$ $\lambda = 0.3$ $\gamma = 0.3$ $\delta = 0.1$ $\phi = 0.1$ $\theta = 0.1$			
1	1.3120	1.0454	1.255
2	1.7270	1.0927	1.5806
3	2.2810	1.1418	1.9977
4	3.0231	1.1929	2.5344
5	4.0214	1.2459	3.2277
Set # 6 $\alpha = 0.15$ $\lambda = 0.35$ $\gamma = 0.315$ $\delta = 0.075$ $\phi = 0.05$ $\theta = 0$			
1	1.0589	1.1420	0.9264
2	1.1162	1.3014	0.8577
3	1.1742	1.4796	0.7936
4	1.2313	1.6777	0.7339
5	1.2863	1.8967	0.6782
Set # 7 $\alpha = 0.15$ $\lambda = 0.35$ $\gamma = 0.32$ $\delta = 0.075$ $\phi = 0.05$ $\theta = 0.1$			
1	1.2377	1.1420	1.0838
2	1.5510	1.3014	1.1918
3	1.9709	1.4796	1.3320
4	2.5436	1.6777	1.5161
5	3.3397	1.8967	1.7609
Set # 8 $\alpha = 0.15$ $\lambda = 0.35$ $\gamma = 0.2$ $\delta = 0.075$ $\phi = 0.05$ $\theta = 0.3$			
1	1.5329	1.1420	1.3423
2	2.4510	1.3014	1.8840
3	4.1133	71.496	2.7800
4	7.2771	1.6777	4.3374
5	13.6551	1.8967	7.2000

In all of our computations, we used K = 10. It may be noticed that if $\eta = \theta - \lambda\phi = 0$ from (20) and (36) it is evident that urbanization has no effect on the amount of agriculture produce, because the amount of food lost due to urbanization has been imported/substituted. Here equations for $p(t)$ and $f(t)$ are decoupled. However N will be affected. If $N = f/p < 1$, there will be a scarcity of food. thus if p increases and f remains constant, there will be less and less food available.

Table 2

t	f/f_0	p/p_0	N/N_0
Set # 1 $\alpha = 0.15$ $\lambda = 0.435$ $\gamma = 0.2$ $\delta = 0.075$ $\phi = 0.15$ $\theta = 0$			
1	1.7013	1.1420	0.9381
2	1.3860	1.3014	0.8745
3	1.1995	1.4796	0.8106
4	1.2512	1.6777	0.7457
5	1.2909	1.8567	0.6806
Set # 2 $\alpha = 0.15$ $\lambda = 0.2$ $\gamma = 0.2$ $\delta = 0.075$ $\phi = 0.1$ $\theta = 0$			
1	1.1092	1.1420	0.9713
2	1.2265	1.3014	0.9425
3	1.3518	1.4796	0.9136
4	1.4842	1.6777	0.8847
5	1.6228	1.8967	0.8556
Set # 3 $\alpha = 0.15$ $\lambda = 0.1$ $\gamma = 0.2$ $\delta = 0.075$ $\phi = 0.01$ $\theta = 0$			
1	1.1312	1.1420	0.9912
2	1.2811	1.3014	0.9843
3	1.4496	1.4796	0.9797
4	1.6401	1.6777	0.9776
5	1.8551	1.8967	0.9781
Set # 4 $\alpha = 0.15$ $\lambda = 0$ $\gamma = 0.2$ $\delta = 0.075$ $\phi = 0$ $\theta = 0$ (NO URBANIZATION)			
1	1.1331	1.1420	0.9923
2	1.2840	1.3014	0.9923
3	1.4550	1.4796	0.9866
4	1.6487	1.6777	0.9834
5	1.8683	1.8967	0.9850

Thus from the point of view of practically all the results given by our model bear strong qualitative agreement with the agricultural economy.

An Example on Pricing. Let us consider a country where wheat is the staple food and at certain initial time, $t = 0$ it is available to 90% population at an affordable price. Thus $N_0 = 0.9$. Let the data set#6, #7 and #8 represent the state of agricultural production, population, urbanization and food import for the next five years. For table of set #6, $\theta = 0$ implying that no wheat is imported; for set #7, $\theta = 0.1$ implying that 10% wheat has been imported per unit of population and per year, and for set #8, $\theta = 0.3$ implying 30% import.

After one year, $t = 1$, if we consider no-import and 10% import, then

$$N_1^1\Big|_{\theta=0} = 0.9264 \times 0.9 = 0.8338$$

$$N_2^1\Big|_{\theta=0} = 1.0838 \times 0.9 = 0.9754$$

Let r_1^1 = price of wheat per unit population per year set by the supplier for $\theta = 0$

= \$150.

Let $r_2^1\big|_{\theta=0.1}$ = \$170.

Then from (56) r^{*1} = price for the consumer

$$= \$170 + \frac{1-0.9754}{0.9754-0.8338} \cdot (170-150)$$

= \$173.48

If instead, 30% is imported and the supplier expects to sell the same amount for \$184, then

$$r^{*1} = 184 + \frac{1-1.3423}{1.3423-0.8338}(184-150)$$

= \$161.11

If it is continued for the next year then

$$r^{*1}\big|_{\theta=0.1} = \$165.17$$

and

$$r^{*1}\big|_{\theta=0.3} = \$158.40$$

Under such conditions, economists of certain countries, especially those who are unionized, prefer to import food, rather than producing themselves. The downside of such a trend is the longterm effect, which could be detrimental as we will notice in a forthcoming article on which we are currently working.

Rate of Inflation. The rate of inflation for this agricultural economy may be approximated by

$$\frac{r^{*,n+1} - r^{*,n}}{t_{n+1} - t_n}$$

If this rate of increase of food price is not balanced by developing the state of overall economic infrastructure, average people will be forced to spend more money for food and less for other industrial products which could result in widespread recession, having a severe global impact.

Thus a high yield in agriculture is an absolute necessity to maintain a stable descent world economy.

9. Conclusion

In engineering and physical sciences, mathematical modeling predict results with both qualitative and quantitative accuracies. This is done because all the parameters in the model could be given with great accuracy by experimental data. But in biological sciences including agriculture, degree of accuracy of data is dependent upon statistical observations and analysis and not all parameters could be properly estimated. Thus, models should be used to make judgements on qualitative predictions and then analyze the available data and check their future trend. The model that is developed in this work should be used likewise. One element which affects agriculture very strongly is the weather pattern. This has not been included directly in the model. It has been done only by adding a general term. All economic predictions must be validated by statistics. This is true for our model too. Finally, our model could be used as an interactive global model to distribute agricultural products among all nations and thereby feeding the hungry on all streets. When it comes to food, global economy should not be ignored. Despite of infinite diversities amongst us, we are one people under one sky, and as such we all must live together helping each other. We must share all our resources with those who are less fortunate. This could be done by the technique of restricted supply for developed well-to-do countries and subsidizing food for the less fortunate by applying the technique of enhanced supply Figure 3, and at the same time advanced technological education should be imparted in the less fortunate world. Instead of saying "China can feed herself" or "India can feed herself" or "Germany can feed herself," we must keep our eyes toward all people of the world and say "we can feed ourselves." That should be the one universal food-ethic which we must respect and act accordingly.

In our model we have attempted to study various aspects of agricultural economics. But the one most dominant aspect is the ethical aspect, where mathematical modeling does not work. In this respect, science must be replaced by sympathy, nationalism must be replaced by humanism which goes far beyond all geographical borders. Feeding the Hungry is possibly the noblest act of human life.

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URBANIZACIJOS IR GYVENTOJŲ SKAIČIAUS AUGIMO IR POVEIKIO AGROEKONOMIKAI MATEMATINIS MODELIAVIMAS

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Santrauka. Gyventojų skaičiaus augimas daro įtaką urbanizacijos procesui ir skatina kultūrinius bei ekonominius pokyčius. Urbanizacija skatina pastatų statybą, kelių tiesimą, prekybos centrų, ligoninių, rekreacinių zonų ir kitų objektų plėtrą, tačiau sukelia ir neigiamų padarinių. Augant gyventojų skaičiui kyla maisto produktų gamybos bei urbanizacijos problemų. Norėdami patenkinti augančio gyventojų skaičiaus poreikius žemės ūkio ir žuvininkystės šakų atstovai turėtų pagerinti maisto produktų tiekimą ir užtikrinti, kad jie atitiktų kokybės reikalavimus, deja, dėl sparčių urbanizacijos tempų tai įgyvendinti darosi vis sunkiau. Straipsnyje keliami aktualūs klausimai, kaip gyventojų skaičiaus augimo, urbanizacijos ir maisto produktų gamybos problemas spręsti darniai ir subalansuotai. Atsakymo ieškoma pasitelkiant matematinius modelius. Pasiūlytas matematinis modelis patvirtina, kad maisto produktų ir jų gamybos galimybių trūkumui, kai gyventojų skaičiaus ir urbanizacijos augimas nėra tinkamai kontroliuojami, turi įtakos darnaus vystymo ir rizikos nustatymo faktoriams.

Tam tikrų matematinio modelio parametrų įvedimas susijęs su gyventojų skaičiaus ir urbanizacijos tempų augimu. Matematinis modelis leido įvertinti šio augimo poveikį maisto produktų gamybai. Modelį sudaro diferencialinių lygčių sistemos. Šios sistemos priemonėmis bandoma susieti interaktyvios ekonomikos sąveikas. Manyta, kad kol gyventojų skaičius auga nepriklausomai nuo urbanizacijos tempų bei žemės ūkio produktų gamybos, urbanizacijos norma priklauso nuo gyventojų augimo, o maisto produktų gamyba priklauso nuo abiejų dedamųjų, t. y. urbanizacijos ir gyventojų. Siekiant išvengti katastrofiško maisto produktų trūkumo gausiai apgyvendintuose regionuose, į maisto produktų gamybos normos apskaitą buvo įtraukta papildoma dedamoji. Tokiam parametrui (dedamajai) galėtų atstovauti maisto pakaitalai arba maisto importas. Matematiškai buvo įvertinta šio papildomo maisto tiekimo šaltinio įtaka. Jei ji išliks kaip auganti dedamoji, galima laukti ir katastrofiškų pasekmių. Šio supaprastinto modelio priemonėmis bandyta priartėti prie kokybinių susitarimų ir realaus pasaulio scenarijų vertinimo. Tolesnis uždavinys sprendžiant šias problemas – rasti statistinius duomenis, pagrindžiančius šių vertinamų parametrų modelio teisingumą ir tinkamumą skirtingiems žemės ūkio ekonomikos scenarijams skirtinguose pasaulio regionuose. Bandoma įvertinti ir prieinamų žemės ūkio produkcijos kainų bei gyventojų skaičiaus augimo santykio pokyčius.

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