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THE DYNAMIC MODEL OF OPTIMAL ECONOMIC GROWTH IN TERMS OF ECOLOGICAL BALANCE: MODELS CONSTRUCTION AND RESULTS ANALYSIS

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Abstract. The article is dedicated to the problem of modeling dynamic ecological-economic models in the case of ecological-economic balance. The nonlinear variant of the model of economic growth considering the ecological balance is proposed and investigated. The mathematical theory of optimal control has been selected for the research of economic growth model.

Keywords: economic growth, sustainable development, ecological balance, optimal control.
JEL classification: O44, E27.

Introduction

Problems of greening of the economy today are ones of the most urgent problems. Human interaction with the environment occurs mainly due to the economic activity that is the main source of pollution. Therefore, the greening and modernization of production and the strategies development of ecologically oriented management for the purpose of the transition to sustainable development are ones of the priority tasks of the theory and practice of decision-making in the economic system of any scale (regional or global). The successful solution of complex problems is impossible without intensive research aimed to study and creation of appropriate methodological tools and methods of finding these solutions and their implementation.

A special place in these studies belongs to economic mathematical modeling as an abstraction of real economic or ecological-economic systems and processes in their mathematical models and, conversely, the transition from analysis of economic and mathematical models to the conclusions about the real economic, environmental and ecological-economic dynamics are efficient and economical methods of research.

Models of economic growth play an essential role in economic and mathematical studies since the 30s of the twentieth century. In particular, E. Domar (Domar, 1957), R. Harrod (Harrod, 2002), P. Samuelson (Samuelson, 1947), R. Solow (Solow, 1956), F. Ramsey (Ramsey, 1928) were engaged in specification on issues of economic growth by developing appropriate models. The development of macroeconomic models allows analyzing, planning and predicting relationships between global economic indicators, which include national income, labor force and production facilities. Macroeconomic growth models show changes in aggregated indicators and provide valuable information about the rate of development in particular sectors of the economy.

There is a huge amount of literature on economic development problem and environmental sustainability analysis (Cairns, 2014; Shieh, 2014; Cherniwchan, 2012; Hui Zuo, 2011; Abou-Ali, 2013). These articles provide construction of overall sustainable development index and investigation of the relationship between natural resource availability, economic growth and environment (Abou-Ali, 2013), investigation of the relationship between current consumption and sustainability improvement (Cairns, 2014), development of an endogenous growth model (Shieh, 2014). All the results show an interlocking relationship between economic growth, environment and sustainable development. Nevertheless, complexity and diversity of the eco-economic systems and sustainable development require further investigation with the aim of new methods construction.

Thus, the aim of the article is modeling of economic growth, taking into account the conditions of the ecological balance of the system, i.e., compliance with sustainable development, and analyses of modeling results.

A large number of constructed models addresses various issues of sustainable development. Among these models, some devoted to mathematical modeling of ecological and economic interactions can be singled out (Onishchenko, 2006; Lyashenko, et al., 2006; Rumina, 2000).

In this article, the nonlinear model of economic growth of an ecologically balanced economy is proposed (Hryhorkiv, 2004). To investigate the model, mathematical theory of optimal control was chosen (Krotov, 1990). The optimal solution must provide information about marginal economic potential, knowledge of which allows setting realistic goals and formulating the problem of managing of the economic system.

The nonlinear model of optimal growth in an ecologically balanced economy

Let the economy consist of basic (material) and secondary (auxiliary) productions. Auxiliary production destroys the pollutants that are formed during the functioning of basic production. Primary production associated with a production, including pollution and auxiliary production, (treatment plant) is concerned only with the destruction of pollutants.

The dynamics of this economy will be described by the following variables:

t is the time variable, $W(t)$ is the industrial consumption (the part of gross production that is recycled), $X(t)$ is the gross product, $Y(t)$ is the final product, $C(t)$ is the unproductive consumption, $I(t)$ are the total investments, $I_b(t)$ are the investments in basic (material) production, $I_z(t)$ are the investments in auxiliary production (treatment plants), $K_b(t)$ is the capital of basic (material) production, $K_z(t)$ is the capital of auxiliary production, $L(t)$ is the manpower, $Z(t)$ is the pollution (e.g., the amount of pollutants in the environment at the moment t), $A_b(t)$ are the amortization charges to basic production assets, $A_z(t)$ are the amortization charges to auxiliary production assets.

To build a model of ecological-economic dynamics, let us form the following basic assumptions:

1. The industrial consumption expenditures are directly proportional to the value of gross production by a factor a $(0 < a < 1)$:

$$W(t) = aX(t) \tag{1}$$

2. The gross production output is determined by the neoclassical production function that is considered to be doubly continuously differentiable and linearly homogeneous:

$$X(t) = F\left(K_b(t), L(t)\right) \tag{2}$$

3. The manpower is exogenous variable with sustainable growth rate $\eta = const$:

$$L(t) = L_0 e^{\eta t} \tag{3}$$

4. The total investment is allocated to primary and secondary production:

$$I(t) = I_b(t) + I_z(t) \tag{4}$$

5. The investments in basic and secondary (auxiliary) production are completely used for the increment of the corresponding capital depreciation:

$$I_b(t) = \dot{K}_b(t) + A_b(t), \tag{5}$$

$$I_z(t) = \dot{K}_z(t) + A_z(t); \tag{6}$$

6. The amortization charges are directly proportional to the corresponding capital value at any time:

$$A_b(t) = \mu_b K_b(t), \tag{7}$$

$$A_z(t) = \mu_z K_z(t) \tag{8}$$

where:

$\propto_b$, $\propto_z$ are amortization coefficients $\left(\mu_b,\, \mu_z \in (0,1)\right)$

7. The criterion of economic development on a fixed time period $[0,T]$ is the maximization of the integrated non-productive consumption:

$$J = \int_0^T e^{-\delta t} C(t)\, dt \tag{9}$$

where $\delta = const > 0$.

8. The economy being considered is developed under stable ecological balance conditions (Crelle, 1988). That is the state when the volume of contamination is independent of the time:

$$Z(t) = Z_{\min} = const \tag{10}$$

Ecological balance (10) is formalized as follows:

$$\alpha(X(t)) - \beta(K_z(t)) = \gamma(Z(t)) \equiv \gamma_0 \equiv const, \tag{11}$$

where $\alpha(X(t))$ is the function of produced pollution, $\beta(K_z(t))$ is the function of eliminated pollution, $\gamma(Z(t))$ is the function of the self-purifying pollution.

Moreover, let us assume that the functions $\alpha(X(t))$ and $\beta(K_z(t))$ are linear:

$$\alpha(X(t)) = \alpha_0 + \alpha_1 X(t), \tag{12}$$

$$\beta(K_z(t)) = \beta_0 + \beta_1 K_z(t), \tag{13}$$

where the constants α_0, β_0, α_1, β_1 and γ_0 play an important role in actual estimation of the influence of the ecological factor on economic development.

Substituting (12) and (13) in (11), the following formula is obtained:

$$\alpha_1 X(t) - \beta_1 K_z(t) = \varepsilon_0, \tag{14}$$

where $\varepsilon_0 = \gamma_0 - \alpha_0 + \beta_0$ (α_0 is the pollution in the absence of economic activity, β_0 is the purification in the absence of economic activity, γ_0 is the natural purification).

The aim of the construction of the economic dynamics model under stable ecological balance (14) reduced to the construction of the capital dynamics equations, which is involved in the basic producing $K_2(t)$.

Let us assign specific indicators:

$$k = \frac{K_b}{L},\ c = \frac{C}{L},\ x = \frac{X}{L} = F(k,1) = f(k),\ F(K_b, L) = Lf(k).$$

Then:

$$\frac{\partial F}{\partial K_b} = f'(k), \quad \frac{\partial F}{\partial L} = f(k) - kf'(k), \quad f'(k) > 0, \quad f''(k) < 0, \quad \lim_{k \to +0} f'(k) = \infty,$$

$$\lim_{k \to \infty} f'(k) = 0.$$

In the capacity of a control parameter, the fraction of non-production consumption is used in the final production $u = \dfrac{C}{Y}$ $(0 \le u \le 1)$.

Let us set the initial and final states for the capital-labor ratio $k(t)$ when $t = 0$ and $t = T$. Let us assume that planning horizon T is final and sufficiently large.

After converting the basic assumptions and entering the specific parameters, the final version of the optimal economic dynamics model under ecological balance will be as follows:

$$\begin{cases} \int\limits_0^T e^{-\delta t} u(t)(1-a) f(k) dt \mapsto \max, \\ \dot{k} = \lambda(t, k(t)), \\ k(0) = k^{(0)}, k(T) \geq k^{(T)}, \\ 0 \leq u(t) \leq 1, \end{cases} \tag{15}$$

where $k^{(0)}$ and $k^{(T)}$ are positive values. Dynamic equation of productive capital takes the following form:

$$\dot{k} = \lambda(t,k) = \left\{ \left[(1-u(t))(1-a) - \frac{\alpha_1}{\beta_1}(\mu_z + \eta) \right] f(k) - (\mu_b + \eta) k + \right.$$

$$\left. + \frac{\mu_z \varepsilon_0 e^{-\eta t}}{\beta_1 L_0} \right\} \cdot \left(1 + \frac{\alpha_1}{\beta_1} f'(k) \right)^{-1}.$$

Thus, model (15) is mathematically an optimal control problem, which aim is to build process $\pi^*(t) = (u^*(t), k^*(t))$, $t \in [0,T]$, that is acceptable and maximizes functional (9), which controls the phase variable (Krotov, 1990).

By determined terms of an optimal process existence $(u^*(t), k^*(t))$, $t \in [0,T]$, the algorithm of the optimal control problems solution was built. This algorithm allows writing all components of the optimal process in an explicit analytic form under certain constraints on the parameters of the model. To control the computational experiments and display the found results of the ecological and economic model (15), a software using Matlab was built.

The algorithm for finding solutions of the model

As a software core, the algorithm for finding a responsible solution to the problem of optimal control is used. This algorithm allows writing all the components of the optimal process in an explicit analytic form and consists of the following steps:

1. Select the type of production function $f(k)$, set the parameters, determine the derivatives $f'(k)$, $f''(k)$.
2. Check the conditions: $f'(k) > 0$; $f''(k) < 0$; $\lim\limits_{k \to +0} f'(k) = \infty$; $\lim\limits_{k \to \infty} f'(k) = 0$.
3. Set the parameters values a, α_1, β_1, η, $\propto_b$, μ_z, ε_0 / L_0, δ (a, $\propto_b$, $\propto_z \in (0,1)$, $\alpha_1 > 0$, $\beta_1 > 0$, $\eta > 0$, $\varepsilon_0 > 0$, $\delta > 0$).
4. Check the condition: $(1-a) - \dfrac{\alpha_1}{\beta_1}(\mu_z + \eta + \delta) > 0$.
5. Solve an equation $\left((1-a) - \dfrac{\alpha_1}{\beta_1}(\mu_z + \eta + \delta) \right) f'(k) - (\mu_b + \eta + \delta) = 0$ for k using a numerical method and find the value of the capital-labor ratio $\tilde{k}$ that defines the main plot of the optimal trajectory.
6. Find the values u_0, u_1 and control the circuit $\tilde{u}(t)$ according to the next equation:

$$\tilde{u}(t) = u_0 + u_1 e^{-\eta t}, \tag{16}$$

where $u_1 = \dfrac{\mu_z \varepsilon_0}{\beta_1 L_0 (1-a) f(\tilde{k})}$, $u_0 = 1 - \dfrac{\alpha_1(\mu_z + \eta)}{\beta_1(1-a)} - \dfrac{(\mu_2+\eta)\tilde{k}}{(1-a) f(\tilde{k})}$.

7. Check the conditions $u_0 \geq 0$; $u_0 + u_1 \leq 1$ according to the determined value $\tilde{k}$.
8. Set an initial state, i.e., time t_0, and an initial value of capital-labor ratio $k^{(0)}$.
9. Set the desired final conditions $t = T$ and $k^{(T)}$.
10. Determine the maximum attainable value of the capital-labor ratio $\hat{k}$, solving an equation $\left((1-a) - \dfrac{\alpha_1}{\beta_1}(\mu_z + \eta)\right)\dfrac{f(k)}{k} - (\mu_2 + \eta) = 0$.

11. Check whether the described dynamic ecological-economic system achieves the planned value of the capital-labor ratio $0 < \max\left(\tilde{k}, k^{(T)}\right) < \hat{k}$.

12. Determine $\bar{k} = \min\left(\tilde{k}, k^{(0)}\right)$ and check the condition $\dfrac{\mu_z \varepsilon_0}{\beta_1 L_0} < \dfrac{\alpha_1}{\beta_1}(\mu_z + \eta) f(\bar{k}) + (\mu_b + \eta)\bar{k}$.

13. If $k^{(0)} < k^{(T)}$, then choose the control circuit $u = 0$ that leads to an increase in capital-labor ratio (otherwise choose $u = 1$).
14. Solving the differential equation $\dot{k}(t) = \lambda(t, k(t))$ with the initial conditions t_0, $k^{(0)}$ and $t > t_0$, determine limiting trajectory $k_\Lambda(t)$.
15. Solving the differential equation $\dot{k}(t) = \lambda(t, k(t))$ with the initial conditions $t = T$, $k^{(T)}$ and $t < T$, determine limiting trajectory $k_\Pi(t)$.
16. Using spline approximation $k_\Lambda(t)$, use a numerical method to solve the equation $k_\Lambda(t) = \tilde{k}$ for t. This solution is a left-handed moment τ^* of switching optimal control.
17. Using spline approximation $k_\Pi(t)$, use a numerical method to solve the equation $k_\Pi(t) = \tilde{k}$ for t. This solution is a right-handed moment τ^{**} of switching optimal control.
18. Develop optimal process according to the next rules:

- selecting the control value under paragraph 13, transfer the system from a state $\left(t_0, k^{(0)}\right)$ for a period of time $\left[t_0, \tau^*\right]$ in a state $\left(\tau^*, k_\Lambda\left(\tau^*\right)\right) = \tilde{k}$;
- selecting the control value $\tilde{u}(t)$, transfer the system from a state $\left(\tau^*, k_\Lambda\left(\tau^*\right)\right)$ for a period of time $\left[\tau^*, \tau^{**}\right]$ in a state $\left(\tau^{**}, k_\Pi\left(\tau^{**}\right)\right) = \tilde{k}$;
- selecting the control value under paragraph 13, transfer the system from a state $\left(\tau^{**}, k_\Pi\left(\tau^{**}\right)\right)$ for a period of time $\left[\tau^{**}, T\right]$ in a state $\left(T, k^{(T)}\right)$.

The above algorithm specifies the procedure for computing the boundaries of time intervals $\left[t_0, \tau^*\right]$, $\left[\tau^*, \tau^{**}\right]$, $\left[\tau^{**}, T\right]$. During these three periods, three different control values must be chosen, which allows an optimal control system transition from the initial state to the final.

Empirical analysis

To find the solution of the nonlinear model of economic growth in an ecologically balanced economy and to analyze simulation experiments, the software using the Matlab language with interactive graphical tools was built. For the simulation, a specific example was used, in which production function was based on the real statistics on Ukraine $f(k) = 0{,}9714k^{0{,}4225}$ (State Statistics Service of Ukraine, 2014).

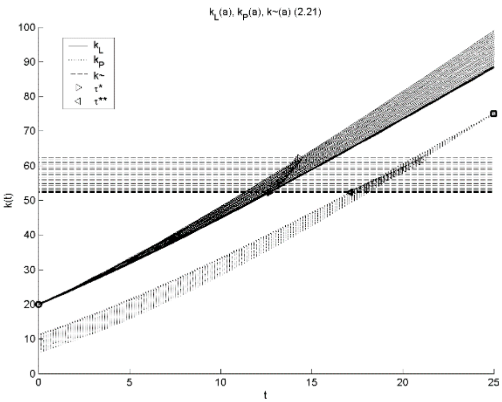
The nonlinear model of optimal growth of an ecologically balanced economy allows planning experiments to study the dependence of optimal solutions and the parameters included in the model (production functions parameters, initial conditions). In connection with the point previously mentioned, a set of experiment management programs was developed. They are designed to detect dependencies $k_\Lambda(t)$, $k_\Pi(t)$, $\tilde{k}$, τ^*, τ^{**}, K^* on parameters included in the modeling equation.

For the model (15), the authors of the present work have investigated how trajectories k_Λ, k_Π, $\tilde{k}$ and points τ^*, τ^{**} vary depending on model parameters a, α_1, δ, $\propto_b$, $\propto_z$, η.

It was found out that due to changes in model parameters within acceptable limits, phenomena could be observed that complicate the practical solution of the control problem and reveal the essence of simulated environmental and economic effects.

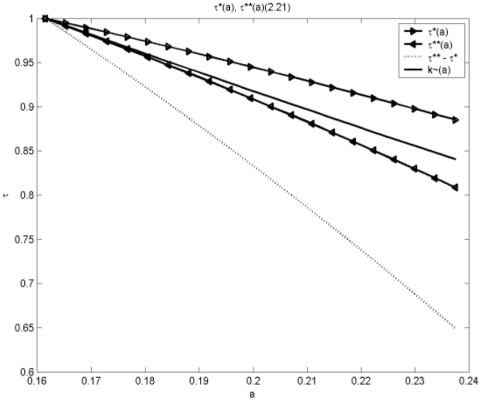
Let us describe the dependence of trajectories and tripping points on the parameter a – the production costs ratio (Figure 1). Figure 1 illustrates the reduction in the capital-labor ratio $\tilde{k}$ that is required for the optimal transition to the final state, as well as reducing the transition time to turnpike of the sustainable $\tilde{k}$. This is evident from the illustrations of several solutions of the model (15) for different values of the parameter a (Figure 2).

Figure 1: The dependence of the model (15) optimal trajectories and production costs ratio a



Source: created by the author

Figure 2: The dependence of τ^*, τ^{**}, $\left(\tau^{**}-\tau^*\right)$, $\tilde{k}$ and the parameter a



Source: generated by the authors using Matlab

Trajectories analysis shows that increasing of the production costs rate reduces the value of the capital-labor ratio $\tilde{k}$ that is required for the optimal transition to the final state, and shift left the points τ^* and τ^{**}. These dependencies $\tau^*(a)$, $\tau^{**}(a)$, $\tilde{k}(a)$ and $\left(\tau^{**}-\tau^*\right)$ are illustrated in Figure 2.

The variation of α_1, η and $\propto_b$ in the ecological-economic model (15) shows the same variation of trajectories k_Λ, k_Π, $\tilde{k}$ and points τ^*, τ^{**} as in the case with the parameter a. The value of the capital-labor ratio $\tilde{k}$ and transition time to turnpike $\left(\tau^{**}-\tau^*\right)$ are reduced.

The variation of the parameter δ has no affects (experiments failed to detect this influence) on the shift of trajectories $k_\Lambda(t)$, $k_\Pi(t)$. Therefore, increasing of δ leads to a shift of the boundary $\left[\tau^*,\tau^{**}\right]$ only through its impact on $\tilde{k}$. Moreover, throughout the acceptable region, δ and $\tilde{k}$ are directly related. The higher consumption is, the lower $\tilde{k}$ and the shorter period $\left[\tau^*,\tau^{**}\right]$ are. The consumption rate affects the conditions of optimal control as well as other indicators of economic efficiency (a, α_1, η, $\propto_b$), with the difference that it does not affect the trajectories $k_\Lambda(t)$, $k_\Pi(t)$.

The $\propto_z$ (amortization coefficient recycling) change shows the same quality dependence of the ecological-economic system behavior as in the case with parameters (a, α_1, η, $\propto_2$), but the passage of turnpike increases.

Conclusion

The constructed nonlinear model of economic growth in terms of environmental and economic balance and developed software using the Matlab language allow effectively carry out an analysis of models that are accompanied by graphic illustrations. The importance of the simulation study lies in the fact that is based on the developed models managed to construct optimal control, optimal economic dynamics trajectory.

The analysis of solutions of the optimal control model (15) shows that in addition to finding the most effective method of transition from the initial state to the final desired state, the model is also suitable for the study of the influence of individual parameters on the conditions of such transition. Parameters that can serve as indicators of efficiency (a, α_1, δ, $\propto_b$, η) have a double impact on the characteristics of optimal control – their increase causes a decrease of $\tilde{k}$ and reduces the "turnpike motion".

The nonlinear model of economic growth can be used to analyze the final state of the economic dynamics trajectory and to make relevant economic decisions. These decisions are concerned with the dynamics of capital and its distribution at a specified level, taking into account ecological and economic balance. The computer monitoring system can be used as a tool for decision support and is the basis for the construction of research complex of environmental and economic effects, particularly in designing business plans of industrial enterprises in terms of ecological balance and developing of state policy in environmental and economic area.

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DINAMINIS OPTIMALAUS EKONOMIKOS AUGIMO MODELIS EKOLOGINĖS PUSIAUSVYROS SĄLYGOMIS: MODELIŲ SUDARYMAS IR REZULTATŲ ANALIZĖ

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Santrauka. Straipsnyje analizuojama dinaminių ekologinių-ekonominių modelių problema siekiant ekologinės ir ekonominės pusiausvyros. Siūlomas ir išnagrinėtas netiesinis ekonomikos augimo modelio variantas atsižvelgiant į ekologinę pusiausvyrą. Ekonomikos augimo modeliui tirti pasirinkta optimalaus valdymo matematinė teorija.

Reikšminiai žodžiai: ekonomikos augimas, tvarus vystymasis, ekologinė pusiausvyra, optimalus valdymas.